Quality standards versus nutritional taxes: Health welfare impacts with strategic firms

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Quality standards versus nutritional taxes: Health and welfare impacts with strategic firms

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Abstract

The goal of this paper is to better understand firms’ strategic reactions to nutritional policies targeting food quality improvements and to derive optimal policies. We propose a model of product differentiation, taking into account the taste and health characteristics of products. We study how two firms react to alternative policies: an MQS policy, linear taxation of the two goods on the market, and taxation of the low-quality good. The MQS and the taxation of the low-quality product are the preferred options by a social planner. If taste is moderately important, the MQS policy is chosen by a populist and a paternalist social planner. If taste is a major component of choice, the populist planner chooses to tax the low-quality product whereas the paternalist planner prefers the MQS policy. Finally, for a paternalist social planner, an MQS-based policy always allows for higher levels of welfare than an information policy alone.

Key words: taxation, MQS, product differentiation, strategic pricing, nutritional policy.

JEL codes:I18, L13, Q18

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1 Introduction

Cardiovascular disease and cancers cause almost two-thirds of the overall burden of disease in developed countries. A large number of these chronic diseases are due to lifestyle-related risk factors, most of which are preventable. In particular, poor dietary habits contribute to these diseases through excessive intakes of salt, carbohydrates, and fats and insufficient intakes of fruits and vegetables. According to the World Health Organization (WHO), combined with actions reducing physical inactivity and tobacco use, preventing diet-related risk factors could lead to an increase in the average life expectancy by three to five years in high-income countries (WHO, 2009).

To address these public health issues, governments and public health agencies have been implementing policies intended to promote preventive behaviors thanks to information campaigns and food product labeling. Reviews of these policies show that they have some positive impacts that, however, remain small, at least in the medium term (Brambila-Macias et al., 2011). In addition, these policies are suspected to increase health inequalities, with less-educated individuals responding less to information policy (Étilé, 2013).

Given the modest impacts of information-based policies, public health agencies are now considering other policies to modify the market environment to facilitate healthier food choices, even by non-health-sensitive consumers. A broad range of instruments have been considered, from price policies to nutrition-related standards (Brambila-Macias et al., 2011). In this context, public health agencies and policy makers urge the food industry to favor a better food environment through changes in the quality and variety of foods and through changes in advertising and marketing (WHO, 2012a).

In the design of policies, it is important to anticipate how food firms will react to nutri-
tional policies because, depending on the design of the policy, firms may amplify or weaken its impacts. A few recent works have begun to consider the strategic price reactions of food firms in response to health and nutrition policies (Bonnet and Réquillart, 2013; Allais et al., 2015; Dubois et al., 2016). Firms can also affect the health outcomes of food consumption through their decisions regarding the quality and variety of foods. In general, the response of firms is based on market segmentation and product differentiation. Nutrition and health claims (such as ‘rich in fibers’, ‘light in sugar’, ‘enriched in vitamins’, etc.) play this role by targeting health-sensitive consumers who have higher willingness to pay for additional health functions in foods. Market shares for those products, however, remain relatively small, at approximately 20%. Regarding the remaining part of the market, the nutritional quality of food is more contrasted (Réquillart and Soler, 2014). For this reason, public health agencies urge the food industry to commit, in individual or collective agreements, to reduce the level of ‘bad’ nutrients in food products. Empirical studies have shown some moves, sometimes quite significant, by some brands or for some nutrients (Webster et al., 2011; Rahkovsky et al., 2012). On the whole, however, the economic and public health literatures tend to consider firms to have weak incentives to enter into a product reformulation strategy intended to improve the nutritional quality of products.

Several explanations have been proposed. A first impediment relies on the cost issue. Indeed, reformulation is likely to affect production costs. For example, changes in ingredients might affect variable costs, and the development of new recipes might require R&D expenditures (Traill et al., 2012). A second reason is based on asymmetry of information between producers and consumers. Smith (2004) suggested that consumers do not really know a product’s quality, even when nutrient fact panels are available on food packages, as
these panels are difficult to understand. This asymmetric information problem results in a ‘lemons-style’ breakdown in the market for processed foods, leading to the ‘McDonald’s equilibrium’ in which low quality covers the entire market. A third reason is related to the ‘addiction assumption’. If the consumption of added sugars or fat leads to addictive behaviors, then firms have strong incentives to continue to market foods high in sugar or fat content (Smith and Tasnadi, 2007). This assumption is plausible but remains controversial, even among neurophysiologists and nutritionists. A fourth explanation relies on consumers’ expectations. Indeed, many studies show that, despite the fact that moderate changes in salt or fat content are not always perceived by consumers, once they know, many consumers reject the reformulated product because they believe that ‘healthier’ means ‘less tasty’ (Raghunathan and Naylor, 2006).

An example of strategic change in the quality of products is found in Moorman et al. (2012). These authors investigated how firms responded to standardized nutrition labels on food products required by the US Nutrition Labeling and Education Act (NLEA). They found that the NLEA had a negative effect on the nutritional quality of labelled brands relative to control brands not required to have a nutrition label. The rationale behind this result is found in the correlation between taste and nutrition attributes and consumer arbitrage between taste and nutrition. If consumers believe that nutrition is negatively correlated with taste and the taste characteristic is more important than the nutrition characteristic in consumers’ choices, then the strategic response of firms to the NLEA is to decrease nutritional quality to avoid discouraging consumption. This response is reinforced by the fact that price is also a key variable for consumers, and more nutritious products are likely to be more costly.
To force food quality improvements, policy makers may use various instruments. A first policy relies on the implementation of minimum quality standards (MQS). The theoretical literature on MQS provides mixed insights. In a simple setting, in which differentiation between products relies on a single characteristic, quality standards seem to be quite efficient (Ronnen, 1991; Crampes and Hollander, 1995). For example, the ban upon trans-fatty acid (TFA) in New York state and in Denmark, where a mandatory maximum content of TFA was implemented in 2004, seem to have had positive impacts (Unnevehr and Jagmanaitė, 2008). In a more complex setting, however, in which products are differentiated along multiple characteristics, setting MQS might be counterproductive, even if the market underprovides quality (Deltas et al., 2013). Other tools such as food taxes can be designed to influence the quality chosen by firms. For instance, the regulator might define a quality threshold. Products that have a quality higher than the threshold are not taxed, whereas products that have a quality lower than the threshold are taxed. Such a policy seems to be efficient, provided that the quality threshold is not too stringent, as a firm prefers to reformulate its product to avoid the tax, leading to positive results for health and welfare (Duvaleix-Tréguer et al., 2012).

The goal of this paper is to better understand firms’ strategic reactions to nutritional policies targeting food quality improvements and to derive a set of optimal policies. We propose a model of product differentiation, taking into account both the taste and health characteristics of products, and use it to assess the health and welfare impacts of taxation and MQS-based policies. An important challenge comes from the need to integrate these two characteristics, both of which affect consumers’ utility. Addressing two dimensions in product differentiation models is difficult. In particular, in a duopoly setting in which
firms first choose the characteristics of products and then compete in prices, it remains very difficult to determine the optimal choice of characteristics by firms in a general setting. To resolve this difficulty, researchers generally impose some restrictions on the choice of characteristics and/or on the heterogeneity of consumers. Thus, to analyze nutritional policies, Duvaléix-Tréguer et al. (2012) designed a duopoly model in which products are differentiated according to two characteristics. They studied the impact of the entry of a firm on this market and assumed that the incumbent firm cannot change the characteristics of its product, thus restricting the complexity of the problem. In addition, they also placed some restrictions on the heterogeneity of consumers. Deltas et al. (2013) explored a duopoly operating in a market with consumers who care about both an environmental attribute and another brand-specific attribute. They developed a two-dimensional differentiation model and assumed that firms cannot choose the brand-specific attribute. They also restricted the heterogeneity of consumers, assuming that consumers all value identically the environmental attribute.

In the model of product differentiation we develop, we consider two mono-product firms competing on price and product characteristics. The products are differentiated along a one-dimensional product characteristic axis (e.g., more or less salty), but the position of a product on this axis may affect consumers’ utility in two ways: through its health impact (the lower the content in salt, the greater the health benefits), on the one hand, and its taste (due to the content in salt), on the other hand. Thus, we take into account the linkages between the nutritional quality of food products and their taste characteristics. This complex relationship between the taste and health characteristics of a product is a key point in the analysis of firms’ strategy.
Using this framework, we compare the impact of three policies - imposing an MQS; setting an excise tax based on the nutrient content of the two products; and setting an excise tax based on the nutrient content of the ‘bad’ product - on consumer demand, prices, product characteristics, a health indicator, and welfare.

We show that firms respond differently to the three instruments, leading to different impacts on market and public health outcomes. Among the three policies we analyze, we find that only the MQS policy and the linear excise tax on the low-quality product are effective in a general sense. The choice, however, between the two depends on the priorities of the regulator and on the consumers resistance to move away from their initial taste preferences. Finally, we show that policies intended to change the food market environment allow for greater health benefits and welfare than policies based solely on information campaigns.

The paper is organized as follows. Section 2 presents the model, Section 3 the benchmark equilibrium and Section 4 the choice of qualities by social planner. Section 5 contains the analysis of firms’ strategic reactions to different policy interventions. In Section 6, we determine and compare the optimal policies chosen by social planners. Section 7 concludes.

2 The Model

2.1 An illustrative example

Let us first consider an illustrative example to make explicit the precise questions we wish to address and to justify the main assumptions of the model. This example concerns public health issues related to salt intake. Elevated dietary salt intake is an established risk factor for high blood pressure and cardiovascular events (He et al., 2013; Hendriksen et al., 2014). Consequently, the WHO has recognized excess dietary salt intake as a global problem and
set a worldwide target of less than 5 g salt per day per person (WHO, 2012b).

To reach this goal, governments are implementing salt-reduction strategies in many countries through information campaigns and interventions designed to improve the nutritional quality of foods. As shown by a recent study in the U.K., these actions may have some positive impacts (Shankar et al., 2013).

On the supply side, significant efforts intended to reduce the salt content of food are underway (Dotsch-Klerk et al., 2015). A remarkable case is the U.K., which has been developing a strategy to progressively reduce population salt intake (He et al., 2013). Thus, in 2005, the UK Food Standards Agency established target levels of salt for each food category. The changes in food quality implemented by food firms contributed to a reduction in the average salt intake at the consumer level (Griffith et al., 2014). This reduction, however, has been smaller than expected, and the average consumer’s salt intake remains above the daily target (Shankar et al., 2013; Webster et al., 2010). Overall, these actions have encouraged sodium reductions in existing food products, but consumer acceptance, cost and complications arising from the use of sodium alternatives remain limitations to food reformulation (Kloss et al., 2015).

Regarding consumers, high salt content generally makes food more palatable. The reduction of salt in foods may then significantly alter the taste of the product and be perceived negatively by consumers. As shown by Bobowski et al. (2015), some subjects with low hedonic sensitivity may respond favorably to salt-reduction strategies and would likely have no difficulty in adjusting to the taste of reduced-salt foods. However, subjects with high hedonic sensitivity disliked reduced-salt foods and would likely have difficulty in adjusting to the taste of reduced-salt foods. In addition, most consumers are not really
aware of the health impacts of high salt intakes. Empirical studies in sensory sciences clearly show that consumers differ not only in salt liking but also in their awareness of the health impacts of salt intake and their motivation to decrease salt intake. For instance, Kenten et al. (2013) conducted a study in the U.K. to analyze consumers perceptions of salt in foods. They showed that most participants were unaware of the advised salt guidelines, unclear about how much they consume, and unsure about the precise connection between high salt intake and negative health impacts. Newson et al. (2013) conducted an international study that shows that one-third of consumers were not interested in salt reduction and that the majority were unaware of health recommendations. Grimes et al. (2009) conducted a survey in Australia that shows that 73% of consumers were unaware of the maximum daily guideline for salt. Just under half of the participants were concerned about the amount of salt in their diet and believed their health would improve if they lowered their salt intake.

On the supply side, voluntary commitments by firms to improve the nutritional quality of foods remain modest. Firms reluctance to decrease the salt content in foods echoes consumers resistance. Preferences for salty taste and misperceptions of the health impact of salt intake may limit consumers willingness to reduce consumption of salt, leading to commercial risks for producers that commit to decrease salt content in foods. Moreover, technological solutions that can be used to compensate for taste modification are imperfect, leading many consumers to appreciate the product less and even, for some of them, to reject it (He and MacGregor, 2009). Finally, removing salt in foods may induce additional production costs: to decrease the salt content in ham without worsening its quality in other respects, firms will need additional processes, and this will make production more
costly. Moreover, for example, a piece of ham with less salt contains less water, leading to a decrease in the product weight, resulting in an increase in per unit cost (He and MacGregor, 2009).

Confronted with the limitations of information campaigns targeting consumers and the modest impact of voluntary commitments by firms, the WHO and some governments are considering more coercive interventions based on quality regulation and fiscal policies. For instance, Portugal and Hungary have implemented taxes on foods with high salt content. In the remainder of the paper, we examine the potential effects of these interventions.

2.2 Main assumptions

We develop a duopoly model of product differentiation. There are two firms $i$ and $j$ producing a product with characteristics $x_i$ and $x_j$ respectively. The characteristic is the product’s content of some nutrient (e.g., salt content). Each firm chooses the content of its product on a $[0,1]$ interval. This characteristic has two effects on the product.

First, it affects the taste of the product. From a consumer perspective, this is a horizontally differentiated characteristic, as some consumers might have different preferences for the taste of the product. We model this as in the standard Hotelling model, and a consumer faces a transportation cost that is a function of the distance between her location (denoted $x$) and the location of the chosen product. We assume a uniform distribution of $x$ over $[0,1]$; when $x$ approaches 1, this means that this consumer prefers a high content of the nutrient.

Second, the characteristic $x_i$ also determines the healthiness of the good. We assume that, as in the salt example, we are in a situation in which consumers have to reduce their intake of the considered nutrient, and we focus on a product category in which policies
are designed to make the products healthier. As the $x_i$ characteristic of product $i$ is its nutrient content, the lower $x_i$ is the better the product is from a health perspective.

From the consumer perspective, this health attribute is modeled as a vertically differentiated characteristic. However, in contrast to the taste attribute, the health impact of the product is not perfectly known by consumers. We instead assume that, as suggested in empirical studies presented above, consumers differ in their awareness of, that is in their beliefs concerning, the health impacts of nutrient intakes. We denote $\lambda$ as the awareness of a consumer and assume that it is uniformly distributed over $[0,1]$. A ‘$\lambda = 1$’ consumer perfectly estimates the health impact of the nutrient intake. A ‘$\lambda < 1$’ consumer imperfectly estimates the probability of health damages and underestimates this health impact. A ‘$\lambda = 0$’ consumer does not perceive any risk induced by the consumption of the considered nutrient.

Note that $x_i$ and thus $(1 - x_i)$ are perfectly known by the consumers. In the case of taste, consumers correctly evaluate the impact on their utility of consuming a product of characteristic $x_i$. Conversely, the impact on health, that is $\lambda(1 - x_i)$, is not well estimated by most consumers because it cannot be simply inferred from the location on the horizontal axis. The consequence is that consumers perception of the health impacts of salt intake may be imprecise and even biased. In fact, most of them underestimate these health consequences.

In sum, from the firm perspective, this model is a one-dimensional model of product differentiation. From the consumer perspective, however, it is a two-dimensional model because, for a given characteristic, a consumer infers two characteristics that have an effect on her utility. A consumer is thus represented by a two-dimensional random variable $(x, \lambda)$. 
We denote $U_c(x, \lambda)$ as the utility that consumer $(x, \lambda)$ obtains when she buys one unit of product $c$, $c \in \{i, j\}$, and write:

$$U_c(x, \lambda) = v - t(x_c - x)^2 + \lambda(1 - x_c) - p_c,$$

with $t$ being the per unit transportation cost. Then, $t(x_c - x)^2$ is the transportation cost as in the Hotelling model. It represents the disutility of not consuming the ideal food that consumer $x$ has to suffer if she buys a product of characteristic $x_c$. The term $\lambda(1 - x_c)$ is the health component of the utility, where $\lambda$ is the consumer’s awareness of health, and $1 - x_c$ represents the healthfulness of product $c$. The closer the food is to 1, the lower the health-related utility a consumer obtains. This part of the model is similar to a Mussa-Rosen model with $(1 - x_c)$ being the quality of product $c$. $p_c$ is the price charged by firm $c$. Finally, $v$ is the intrinsic utility that consumers obtain from buying this food. As the two products differ only in their content of the specific ingredient, $v$ is the same for the two products. We assume that $v$ is large enough that the market is covered. In the following, we denote by $U_i$ and $U_j$ the utility a consumer receives when buying one unit of a product from firm $i$ and $j$, respectively. Without loss of generality, we assume that $x_j > x_i$. Firm $i$ produces the ‘healthy’ product, and firm $j$ produces the ‘unhealthy’ product. We will also refer to these products as high-quality and low-quality products, respectively.

By equating $U_i$ and $U_j$, we obtain the indifference line along which the consumers are indifferent between consuming product $i$ and consuming product $j$. It is given by

$$\lambda = 2tx - t(x_j + x_i) - \frac{p_j - p_i}{x_j - x_i}. \quad (2)$$

Note that the slope of the indifference line is $2t$. If $t \geq 1/2$, then the slope of the
indifference line is greater than (or equal to) 1, which means that taste weights more than the consumers’ perception of health impacts. If \( t < 1/2 \), the opposite holds. As discussed above, consumers place a higher weight on taste than health in their food choices. As a consequence, in the following, we concentrate on the case in which \( t > 1/2 \), as taste plays an important role in consumers’ choice. Consumers located to the left of the indifference line buy from firm \( i \), and consumers located to the right of this line buy from firm \( j \). It is easy to deduce the demand \( D_i \) and \( D_j \) faced by firms \( i \) and \( j \), respectively. When consumer \((0,0)\) buys product \( i \) and consumer \((1,1)\) buys product \( j \), we have

\[
D_i = \frac{1}{2t}[t(x_i + x_j) + \frac{p_j - p_i}{x_j - x_i} + \frac{1}{2}],
\]

\( (3) \)

\[
D_j = 1 - D_i = \frac{1}{2t}[\frac{4t - 1}{2} - t(x_i + x_j) - \frac{p_j - p_i}{x_j - x_i}].
\]

\( (4) \)

We also define the demand functions when consumer \((0,0)\) buys product \( j \) or when consumer \((1,1)\) buys product \( i \), as they are useful for fully characterizing the equilibrium (see the Appendix). \(^1\)

From the supply side, modifying the nutrient content of food is costly. Following Duvaleix-Tréguer et al. (2012), we assume that the marginal cost of production is a quadratic function of the product’s healthfulness, that is, \( 1 - x_c \) for a firm producing a product of characteristic \( x_c \). \(^2\) We write

\[
c(x_c) = \frac{(1 - x_c)^2}{2}.
\]

\( (5) \)

\(^1\)Technically, with \( t < 1/2 \), it is more convenient to define the demand depending on the value of \( \lambda \) when \( x = 0 \) or \( x = 1 \). Consumers located under the indifference curve consume product \( j \), and consumers located above the indifference curve consume product \( i \). As before, we also obtain three cases for demand.

\(^2\)With a linear form, in the standard case in which consumer \((0,0)\) chooses product \( i \) and consumer \((1,1)\) chooses product \( j \), we do not obtain equilibrium in which \( x_i \) and \( x_j \) are interior solutions on \([0,1]\).
The profit functions of a firm is written as follows:

\[ \Pi_c = [p_c - (1 - x_c)^2]D_c, \quad c = (i, j), \]

which is the product of the per unit mark-up and the quantity demanded.

2.3 Health index and welfare

The goal of the paper is to analyze how policies affect the equilibrium prices, the characteristics of the products, and their impact on health and welfare. We now define the health index and welfare.

As explained above, \( x_c \) is the content of a product \( c \) with respect to a nutrient that adversely affects health. The lower \( x_c \) is, the better it is from a health perspective. Then, \( (1 - x_i) \) and \( (1 - x_j) \) are indicators of the healthfulness of products \( i \) and \( j \), respectively. As each consumer consumes one unit of either of the two products, the health status of each individual is directly related to the quality (nutrient content) of the chosen product. In epidemiological models, the impact on health of a change in the consumption of a nutrient is evaluated via a relative risk index (RR). This index links a change in the consumption of the nutrient by a consumer to a change in the probability of contracting a disease. To integrate this issue into our analysis, we define the health index of a consumer consuming a product located in \( x_c \) by \( h_c = (1 - x_c) \). By doing so, we consider the simple case in which the RR does not vary with the initial intake.\(^{3}\)

As we consider public health issues, we define the aggregate index over the market.

\(^{3}\)A more general case would be to assume that the impact of a given change in consumption does depend on the initial level of consumption. In our case of an ‘unhealthy’ nutrient, this would mean that a given decrease in the consumption of the nutrient has a greater impact for a consumer who already has a high level of consumption (that is, who consumes product \( j \)) than for a consumer who has a low level of consumption (that is, who consumes product \( i \)).
quality of the marketed products and the market shares of the different products. Then, we consider the following public health index $h_{ij}$:

$$h_{ij} = (1 - x_i)D_i + (1 - x_j)D_j. \quad (7)$$

The definition of welfare also needs some discussion. Social welfare is the sum of producer surplus, consumer surplus, and tax revenues if there are any tax revenues. The issue is related to how the social planner evaluates the health impact of consumption. As explained above, we assume that consumers differently estimate the health impacts of nutrient intakes. The parameter $\lambda$ is consumer awareness regarding health. A consumer with a low value of $\lambda$ underestimates the health impact of consumption, whereas a consumer with a high value of $\lambda$ almost exactly estimates the health impact. This assumption is similar to the distinction proposed by Salanie and Treich (2009) regarding citizens beliefs concerning water supply contamination. If these beliefs are biased (compared to the real risk perfectly known by the social planner), then it is possible to consider two types of social planners.

A first case corresponds to a populist social planner who evaluates the consumer surplus on the basis of the utility function of consumers (we denote the associated level of social welfare SW1). Such a social planner maximizes social welfare based on citizens beliefs regarding the health impact of food intake. He assumes that the health damages supported by $\lambda < 1$ consumers are the result of their own perception, and a utilitarian maximization is required. A second case corresponds to a paternalistic social planner who fully integrates the health impact of consumption (Cremer et al., 2012). In this case, the social planner is

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4This objective function could be that chosen by the ministry of health, whereas the government as the whole would maximize the standard welfare as the populist social planner does. The ministry of health places significant weight on health, whereas the government as a whole adopts a more balanced view. For
paternalist and maximizes social welfare based on his own beliefs $\lambda = 1$. To some extent, he acts in the place of consumers by taking into account their health damages. The consumer surplus is then evaluated by the social planner on the basis of a modified utility function.\(^5\)

From the social planner’s perspective, a consumer who consumes product $i$ receives the following utility:

$$U'_i = v - t(x_i - x)^2 + (1 - x_i) - p_i. \quad (8)$$

The utility now includes the ‘true’ impact of consumption on health (we denote the associated social welfare $SW2$).

### 3 Benchmark Equilibrium

To analyze competition between the two firms, we assume that they play a two-stage game. In the first stage, firm $i$ and firm $j$ simultaneously choose the characteristic of their products ($x_i, x_j$). After they and consumers observe their choices, they compete à la Bertrand and simultaneously choose prices ($p_i, p_j$). Consumers then make their choice and profits are realized. The design of the game is standard and reflects the idea that the choice of the characteristic of a product is a long-term decision whereas the choice of a price is a short-term decision.

To solve the two-stage game, we determine the perfect equilibrium by backward induction. We determine first the Nash price equilibrium in the sub-game given the characteristics $x_i$ and $x_j$. Then, we determine the optimal characteristics of product, denoted $x^B_i$.

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\(^5\)Note that we assume that all consumers under-estimate the health impact, as stated in many surveys (Newson et al., 2013).
and $x_j^B$.

At the second stage, assuming that consumer (0,0) buys product $i$ and consumer (1,1) buys product $j$, prices are given by

$$p^*_i(x_i, x_j) = \frac{1}{6}[2(1 - x_i)^2 + (1 - x_j)^2 + (x_j - x_i)(1 + 2t(2 + x_i + x_j))], \quad (9)$$

$$p^*_j(x_i, x_j) = \frac{1}{6}[(1 - x_i)^2 + 2(1 - x_j)^2 + (x_i - x_j)(1 + 2t(-4 + x_i + x_j))]. \quad (10)$$

From (9) and (10), we deduce the profits of firm $i$ and $j$, $\pi_i(p^*_i(x_i, x_j), p^*_j(x_i, x_j))$ and $\pi_j(p^*_i(x_i, x_j), p^*_j(x_i, x_j))$, which are maximized wrt $x_i$ and $x_j$, respectively. The equilibrium locations in the first-stage game are given by

$$x_i^B = \frac{1 - t}{2(1 + 2t)}, \quad (11)$$

$$x_j^B = \frac{1 + 5t}{2(1 + 2t)}. \quad (12)$$

Note that (11) and (12) impose $t < 1$. In the Appendix, we provide detailed computations that show that the existence and the unicity of equilibrium depend on $t$.

When $t$ is small ($t < \bar{t} \simeq 0.589$), that is when taste plays a relatively small role compared to health, two equilibriums may exist. One equilibrium is such that $x_i = 0$, and the other is such that $x_j = 1$. In the first case, the quality of product $i$ is maximal and few consumers buy product $i$, that is, those who both like a product with a low content of the nutrient and are aware of the health impact of consuming such a product. All other consumers buy the low-quality product, some because they are unaware of the health impact of doing so, and some because they enjoy eating a product with a high content of the nutrient. In the second equilibrium, $x_j$ is set at the maximum and few consumers buy the product. Those consumers are unaware of the health impact of doing so and prefer a product with a high
content of the nutrient. All other consumers buy the high-quality product, some because
they are aware of the health impact of doing so, and some because they enjoy eating a
product with low content of the nutrient.

When \( t \) is higher (\( t > \bar{t} \approx 0.589 \)), we obtain a unique equilibrium in which the optimal
characteristics of products \( i \) and \( j \) are in the range of possible locations (that is, \( 0 \leq x_i \leq x_j \leq 1 \)). Given the goals of this article, this case is much more interesting, as it corresponds
to situations in which \( t \) is quite large and thus in which consumers are more reluctant to
move away from their preferred location in taste. As suggested above, this is meaningful,
as it is well known that taste plays a major role in food choices. For this reason, in the
remainder of the paper, we limit our analysis to the following case:

\[
\bar{t} \approx 0.589 \leq t \leq 1.
\] (13)

We provide in Table 1 a full characterization of the equilibrium in function of the
parameter \( t \). In equilibrium, firms choose characteristics that are symmetric, that is,
\( x_i^B = 1 - x_j^B \). The profits of both firms are equal. This comes from the fact that the
absolute markups of producers \( i \) and \( j \) are equal (\( p_i^B - c(x_i^B) = p_j^B - c(x_j^B) = \frac{3t^2}{1 + 2t} \)) and
demands are equal.

We now discuss the comparative statics in equilibrium. When \( t \) increases, and assuming
that the characteristics of both products are given, the demand faced by each producer
becomes less elastic. Thus, for a firm, a decrease in its own price, assuming that the price
of the other firm is given, now attracts fewer consumers. Facing a less elastic demand, each
producer has an incentive to increase its price, leading to a higher price in equilibrium.\(^6\)

\(^6\)Technically, when qualities are given, the best reply functions \( p_i(p_j) \) and \( p_j(p_i) \) are upward sloping.
That is, prices are strategic complements. Moreover, in a \( \{p_i, p_j\} \) space, \( p_j(p_i) \) shifts upward when \( t \)
\[
\begin{align*}
x_i^B & = \frac{1-t}{2(1+2t)} & \downarrow \\
x_j^B & = \frac{1+5t}{2(1+2t)} & \uparrow \\
P_i^B & = \frac{1+10t+49t^2+48t^3}{8(1+2t)^2} & \uparrow \\
P_j^B & = \frac{1-2t+25t^2+48t^3}{8(1+2t)^2} & \uparrow \\
D_i^B = D_j^B & = \frac{3t^2}{2(1+2t)} & - \\
\pi_i^B = \pi_j^B & = \frac{3t^2}{2(1+2t)} & \uparrow \\
\chi_j^B - x_i^B & = \frac{3t}{1+2t} & \uparrow \\
\hat{h}_{ij}^B & = \frac{1}{2} & - \\
CSB & = \frac{9+8t-170t^2}{48(1+2t)} & \text{nm} \\
SW_1^B & = \frac{9+8t-26t^2}{48(1+2t)} & \text{nm} \\
SW_2^B & = \frac{15+32t-26t^2}{48(1+2t)} & \text{nm} \\
\end{align*}
\]

\text{nm: not meaningful.}

Table 1: Characterization of the benchmark

However, each firm has also an interest in adjusting the location of its product. Thus, when \( t \) increases, a consumer is more likely to purchase the product near her own location, rather than the more distant product. This allows producers to soften competition by moving away from one another, which allows for a further price increase.\(^7\) In addition, the marginal costs of producers are affected by the change in the characteristics of their products. Producer \( i \) increases the quality of its product, generating an increase in the marginal cost increases, while \( p_i(p_j) \) shifts downward, leading to an increase in prices in (second-stage) equilibrium.\(^7\)

\(^7\)Technically, integrating the second-stage price equilibrium, the best reply functions \((x_i(x_j) \text{ and } x_j(x_i))\) are upward sloping. That is, qualities are strategic complements. When \( t \) increases, in a \( \{x_i, x_j\} \) space, both \( x_j(x_i) \) and \( x_i(x_j) \) shift downward. Given the respective shifts, this leads to a lower \( x_i \) and a higher \( x_j \).
and thus in price. Producer $j$ faces a different situation. Its marginal cost of production decreases as a result of the decrease in the quality of its product (that is, $x_j$ increases). The effect, however, of softened competition on price is larger than that of the marginal cost change, and as a consequence, the price of product $j$ increases. This also explains why the price increase for product $j$ is lower than that for product $i$. As in a standard location model, firms’ profits increase with $t$. It is worth mentioning that in this model both strategic variables, namely the characteristic of the product and price, are strategic complements.

When $t$ increases, the market shares in equilibrium remain constant ($D_i = D_j = 1/2$). However, even if the market shares do not change, some consumers have modified their choices. Given the qualities and prices in equilibrium, the indifference curve is written $\lambda^B(t) = \lambda_t = 2tx - t + 1/2$, as we have $x_i^B + x_j^B = 1$ and $(p_j^B - p_i^B)/(x_j^B - x_i^B) = -1/2$.

Consumer (1/2, 1/2) belongs to the indifference curve $\forall t$. When $t$ increases, in equilibrium, the indifference curve rotates counterclockwise around the point $x = 1/2, \lambda = 1/2$. Some consumers with $\lambda < 1/2$ who were consuming product $j$ (as they were located on the right of the indifference curve) now consume product $i$ (as they are now located on the left of the new indifference curve). Similarly, some consumers with $\lambda > 1/2$ who were consuming product $i$ (as they were located on the left of the indifference curve) now consume product $j$ (as they are now located on the right of the new indifference curve). Because the rotation is around the point $x = 1/2, \lambda = 1/2$ market shares do not change.

---

8Note that, if prices and qualities were not adjusted to the change in $t$ (that is they would still be set at the optimum level for $t$ rather than for $t + \Delta t$, with $\Delta t$ the positive change in $t$), we get a similar move in the indifference curve. This is because, at the ‘unchanged’ prices and qualities, we have $x_i^B + x_j^B = 1$. Then, from (2), the indifference curve can be written as $\lambda = \lambda_t + 2x\Delta t - \Delta t$. The point $x = 1/2, \lambda = 1/2$ still belongs to the indifference curve.
4 Optimal qualities for the social planner

As a reference for the analysis of the impact of policies, we now define the choice of qualities by a social planner. The populist social planner chooses the optimal qualities and the demand allocation that maximize social welfare, given by the following:

\[
SW_1 = \int_{x_0}^{x_1} \int_0^1 U_i d\lambda dx + \int_{x_0}^{x_1} \int_{\lambda(x)}^1 U_i d\lambda dx \\
+ \int_{x_1}^{x_0} \int_0^1 U_j d\lambda dx + \int_{x_0}^{x_1} \int_{\lambda(x)}^1 U_j d\lambda dx \\
- c(x_i) \frac{x_0 + x_1}{2} - c(x_j) [1 - \frac{x_0 + x_1}{2}],
\]

(14)

with \(x_0\) being the location of the indifferent consumer with \(\lambda = 0\) and \(x_1\) being the location of the indifferent consumer with \(\lambda = 1\). Social welfare is written as the sum of consumer surplus minus the production costs. The first two terms in (14) correspond to the surplus of consumers who consume product \(i\), the two following terms correspond to the surplus of consumers who consume product \(j\), and the last two terms are the production costs. The optimal qualities are given by

\[
x_i^{SW_1} = \frac{-1 + 12t + 12t^2}{24t(1+2t)},
\]

(15)

\[
x_j^{SW_1} = \frac{1 + 12t + 36t^2}{24t(1+2t)},
\]

(16)

see details in the Appendix.

It is easy to show that \(x_i^B < x_i^{SW_1}\) and \(x_j^B > x_j^{SW_1}\). Firms have an incentive to differentiate the products to soften price competition. For the social planner, there is no reason to do so, and the distortions come from transportation costs and choices of qualities. The social planner’s choice tends to limit the wasted transportation costs and thus entail
locations corresponding to less differentiated qualities. It should be acknowledged that, in this model in which the market is covered, a change in both prices which leave the market shares unchanged, does not affect the social welfare.

A paternalistic social planner maximizes social welfare using a modified utility function (Eq. 8). The optimal qualities are given by:

\[ x_{SWi}^2 = \frac{1 + 12t^2}{24t(1 + 2t)} = x_{SWi}^1 - \frac{12t - 2}{24t(1 + 2t)}, \]  
\[ x_{SWj}^2 = \frac{-1 + 36t^2}{24t(1 + t)} = x_{SWj}^1 - \frac{12t + 2}{24t(1 + 2t)}. \]  

Given (13), we have \( x_i^B < x_{SWi}^2 < x_{SWi}^1 \) and \( x_j^B > x_{SWj}^1 > x_{SWj}^2 \). As before, the social planner chooses less differentiated products than firms do. Compared with the populist social planner, the paternalistic social planner chooses higher qualities for both products. This is intuitive because this social planner assigns greater importance than the populist social planner to the health component in the utility function of consumers.

5 Strategic Responses to Policy Interventions

We now analyze the impact on prices, product characteristics, health, consumer surplus, and welfare of alternative policy interventions. We analyze first the impact of an MQS policy. The MQS requires firms operating in the market to comply with a certain quality standard. In our context of an ‘unhealthy’ nutrient, the MQS is defined as the maximum content of the ‘unhealthy’ nutrient content of a product. This type of policy was defined in the case of TFA the use of which was progressively prohibited in different countries (Unnevehr and Jagmanaite, 2008). That is, the maximum TFA content of food products

\[ x_0 \text{ and } x_1 \text{ are determined using the consumer utility function as consumers make their choices using this utility function.} \]
was progressively decreased thanks to an MQS policy. Then, we analyze the impact of excise taxes. We design two cases. We consider first a case in which both products are taxed as a function of their nutrient content. This type of tax was introduced in 2011 in Denmark. The tax targeted saturated fats, and was specified in DK/kg of saturated fat in the product (Jensen and Smed, 2013).\footnote{The tax applied to a broad range of products, such as meat, dairy products, animal fats, and vegetable oils. The tax only applied to the products that had a content level of saturated fat that was higher than 2.3%. For many markets, however (e.g., the market for oil), all products were taxed, as it was not possible to completely remove saturated fats from those products.} The second version of the excise tax is to consider that the tax only applies to the low-quality product. In practice, this means that some products are taxed while others are not. We find examples of such taxes in the soft drink market. For example, Hungary introduced a tax on soft drinks that contain more than 80g/l of added sugar (ECORYS, 2014). In practice, it means that diet products are not taxed whereas sugary products are.

5.1 Minimum Quality Standard (MQS)

We assume that the regulator imposes a requirement that the nutrient content of the products on the market should be lower than some threshold $\bar{x}$. The MQS affects the choice of characteristics only if $\bar{x} < x_j^B$. Stage 2 of the competition game is not modified, and (9) and (10) apply because they define the price equilibrium at given product characteristics. Stage 1 of the game is modified as the choice of firm $j$ is constrained: we have $x_{j}^{S} = \bar{x}$.\footnote{We denote with a subscript $S$ the values of variables in equilibrium under the MQS scenario.}

Given that firm $j$ chooses location $x_{j}^{S}$, the optimal choice for firm $i$ is:

$$
x_{i}^{S} = \frac{1 - 4t}{3(1 + 2t)} + \frac{\bar{x}}{3} = x_{i}^{B} + \frac{\bar{x} - x_{j}^{B}}{3}.
$$

When firm $j$ faces a constraining MQS, firm $i$ responds by lowering the nutrient content of its product, that is, by increasing the quality of its product. We formulate in proposition...
1 the main implications of the MQS policy.

Proposition 1. Under an MQS policy that restricts the choice of the characteristic of the low-quality product, and relative to the benchmark case:

• The quality of the high-quality good increases, and the differentiation in the market decreases;

• The price of the high-quality good decreases, whereas the price of the low-quality good increases;

• The high-quality producer is worse off, and the low-quality producer is better off;

• The health index, the consumer surplus, and welfare increase.

Proof. See the Appendix

When the choice of quality by firm $j$ is constrained and thus firm $j$ increases the quality of its product, the high-quality firm also raises the quality of its product, as qualities are strategic complements. As shown in (19), the change in $x_i$ is smaller than that in $x_j$; the differentiation between the two products decreases, leading to tougher competition. The impact on prices depends on two opposite effects. On the one hand, the increase in quality increases the marginal costs of production and then prices according to (9) and (10). On the other hand, the reduced differentiation leads to more intense price competition. The net effect of these two opposite effects is positive for firm $j$; that is, the price of product $j$ increases, whereas it is negative for firm $i$; that is, the price of product $i$ decreases. Firm $j$’s price and demand increase, along with a decrease in firm $i$’s price and demand level. As a consequence, firm $i$ is worse off and firm $j$ is better off. The health index increases due
to the improvement in the quality of both products. This effect dominates the negative impact (on health) of the shift in demand. Thus, the indifference curve (2) shifts to the left. As a consequence, some consumers switch from the high-quality good to the low-quality good. We also find that consumer surplus increases, and social welfare, whether it is defined by a populist or a paternalistic social planner, increases. In this setting in which the market is covered, price distortions due to market power do not affect welfare directly, as this corresponds to a transfer between consumers and producers. The distortion that is reduced is the distortion due to the change in the characteristics of the products. The increase in the quality of product \( j \) (that is, a lower \( x_j \)) decreases the distortion on product \( j \), as \( x_j^B > x_j^{SW1} > x_j^{SW2} \). Conversely, the increase in the quality of product \( i \) increases the distortion on product \( i \), as \( x_i^B < x_i^{SW1} \). This latter negative effect is, however, lower than the positive one due to the increase in the quality of product \( j \).

These results are in line with those of Ronnen (1991) and Crampes and Hollander (1995) who analyzed the impact of imposing an MQS in a duopoly framework with vertical product differentiation and in which firms first choose the quality of their products and then compete on price. Both papers found that the MQS acts as a commitment device for firm \( j \), providing this firm with a first-mover advantage. As a consequence, firm \( j \) is better off and firm \( i \) is worse off. The papers by Ronnen (1991) and Crampes and Hollander (1995) differ with respect to the cost of quality. In Ronnen (1991), an increase in quality has an impact on fixed costs, whereas in Crampes and Hollander (1995), it has an impact on variable costs, as is the case in our framework. Interestingly, in Crampes and Hollander (1995), the qualitative results depend on the quality adjustment of product \( i \). When the quality of product \( i \) increases less than that of the low-quality product, which is the case
in our model, then all qualitative results we have are similar to their results.

However, our results regarding the impact of MQS on product quality differ from those in Deltas et al. (2013). That paper considers a duopoly operating in a market with consumers who care about both an environmental attribute, which could be regarded as the quality attribute, and another brand-specific attribute. With respect to the brand-specific attribute, the firms are assumed to locate at the two ends of a unit interval. With respect to the environmental attribute, consumers have the same willingness to pay for products’ greenness. In their setting, there is only consumer heterogeneity in terms of brand-specific horizontal attributes. Additionally, the horizontal attribute and the vertical attribute are independent. Using this framework, the authors found that firms’ environmental qualities are strategic substitutes. Then, the implementation of an MQS leads the high-quality firm to decrease the quality of its product, which is the opposite of our results. In our setting, on the contrary, product qualities are strategic complements and as a result, an increase in the standard leads to the augmentation of both products’ qualities.

5.2 Excise tax on both products

To penalize the use of the ‘unhealthy’ nutrient, the regulator imposes a tax on both products. To penalize the low-quality product more than the high-quality product, we consider a linear excise tax with rate $f$. That is, a product of characteristic $x_c$ faces a tax $f \cdot x_c$. The profits of firms are given by

\[
\Pi_i = (p_i - (1 - x_i)^2 - f x_i)D_i \tag{20}
\]

\[
\Pi_j = (p_j - a(1 - x_j)^2 - f x_j)D_j \tag{21}
\]
The tax acts as an increase in production costs. We formulate in proposition 2 the main implications of the excise tax policy.

**Proposition 2.** Under an excise tax proportional to the nutrient content of both products, and relative to the benchmark case:

- The content of the taxed nutrient decreases for both products by the same amount;
- The prices of both products increase;
- The profits of producers remain constant;
- The health index increases, the consumer surplus decreases, the welfare evaluated by a populist social planner decreases, and the welfare evaluated by a paternalistic social planner increases as long as the tax rate is not too high.

**Proof.** See the Appendix

To limit the impact of the tax, firms choose higher quality for their products. As a consequence of the tax and the increased qualities, prices increase. Moreover, because the tax is linear, the impact on the marginal cost of quality is identical for both firms (= $f$). This explains why the content of the taxed nutrient decreases by the same amount for both products. In addition, the markups remain constant, as do the market shares. The consequence is that profits do not change. Those adjustments are a consequence of the assumption regarding to market coverage. Thus, the cost increase, which both firms face, can be transmitted to the consumers without losing consumers. The adjustments in price and qualities are such that with linear taxes, the indifference curve is not affected. Because market shares do not change and the quality of both products increases, the health index
increases. However, consumer surplus decreases, as the tax is equivalent to a cost increase transmitted to consumers. Social welfare, defined by a populist social planner, decreases. On the one hand, the distortion affecting the low-quality product decreases, but on the other hand, the distortion affecting the high-quality product increases. The overall effect on welfare is negative. If defined by a paternalistic social planner, however, welfare increases. Thus, the paternalistic social planner values more the health impact of consumption. The increase in the quality of both products explains the increase in the welfare evaluated by the paternalistic social planner.

A variant of this policy is to design a bonus malus taxation policy (maintaining the characteristic that the tax/subsidy scheme is linear with respect to the content of the ‘unhealthy’ nutrient). That is, products that have a level of content of the ‘unhealthy’ nutrient that is lower than some threshold benefit from a subsidy. Conversely, products that have a level of content of the ‘unhealthy’ nutrient that is larger than the threshold are taxed. The idea is to acknowledge that the consumption of most nutrients is not ‘bad’ per se but rather that it is their excessive consumption that has negative health impacts. Formally, rather than defining the tax by $f \ast x_c$ for a product of characteristic $x_c$, the tax is defined as $f \ast (x_c - \tilde{x})$. This is thus a combination of the previous scheme and a fixed subsidy to the firm. In our model, the qualities in equilibrium are not affected by a fixed subsidy (applied to both products) or a fixed tax. Prices change by the amount of the per unit subsidy, and hence absolute markups do not change. Other variables are not affected. We only have a transfer between consumers and taxpayers. Thus, all of the results presented in the case of a linear excise tax on the two products also apply to this type of bonus malus policy.
5.3 Excise tax on the low-quality good

An alternative policy is to discourage the consumption of the low-quality good by imposing a tax on this product. We maintain the same scheme as above, that is a linear excise tax, but apply it to the low-quality product only. Thus, product $j$ faces a tax $f \times x_j$. The profit of firm $i$ is given by (6) and the profit of firm $j$ is given by (21). The tax acts as an increase in the production costs of firm $j$ only. We formulate in proposition 3 the main implications of the excise tax policy.

**Proposition 3.** Under an excise tax proportional to the nutrient content of the low-quality product, and relative to the benchmark case:

- The quality of the high-quality product decreases; the quality of the low-quality product increases as long as the tax rate is not too high;

- The price of the low-quality product increases, whereas the price of the high-quality product decreases;

- The high-quality firm is better off and the low-quality firm is worse off;

- The health index increases, the consumer surplus decreases when the tax is ‘small’ (it can increase for some higher level of tax, depending on $t$), and welfare increases.

**Proof.** See the Appendix

With the tax, firm $j$ faces a cost increase that is proportional to the nutrient content of its product. This provides this firm with an incentive to improve the quality (that is,
to lower the nutrient content $x_j$ of its product. The response of firm $i$, facing a less competitive firm (because its product is taxed), is to reduce the quality of its product (that is, to increase $x_i$). The change in the price of product $i$ results from three elements: a reduction in the cost due to the change in quality, a less differentiated market meaning tougher competition, and an opposite effect, which is facing a less competitive firm. On the whole, the first two effects dominate the third, and hence the price of product $i$ decreases. The change in the price of product $j$ results from two elements: an increase in the cost due to both the taxation and the increase in quality and an opposite effect resulting from a less differentiated market. The first effect dominates the second, and hence $p_j$ increases. The changes in profits are rather intuitive: the taxed firm is worse off, and the other firm is better off.

The change in the health index results from three effects, the first two having a positive impact and the third having a negative impact. First, the quality of product $j$ increases. Second, the indifference curve (2) shifts to the right. As a consequence, some consumers switch from the low-quality good to the high-quality good. Third, the quality of product $i$ decreases. The impact of the first two effects dominates the impact of the third, explaining the increase in the health index with the tax. Consumer surplus decreases as a consequence of taxation, which in this context is fully transmitted to the consumer. It should be noted, however, that at higher levels of taxation, an increase in the tax level might have a (marginal) positive impact on consumer surplus. Welfare increases as a consequence of the change in quality. As explained above, distortions in this model come from the location of the product. In this scenario, both the distortions with respect to product $i$ and product $j$ are reduced. Then, welfare increases. It is, however, interesting to note that the quality
of the low-quality good reaches a maximum for \( f = \frac{1}{2}(3t - \sqrt{t(1+2t)}) \). If the tax rate is higher than that value, then firm \( j \) no longer has sufficient incentives to continue to increase the quality level. On the contrary, it begins to decrease the quality level of its product. However, compared to the benchmark, the quality of product \( j \) is still higher.\(^{12}\)

This is the result of both the increased competition from firm \( i \), as the quality of product \( i \) is still increasing, and the increase in the tax rate. For firm \( j \), when the tax rate is high, it becomes more profitable to lower the quality, thus relaxing competition and decreasing production costs.

### 5.4 Robustness of results

An important assumption of the model relies on the link between the taste of the product and its healthiness. In our case, based on the example of salt in foods, we assume a full link between the two. Then, the nutrient content of the product determines both its taste and its healthiness. An alternative case is when there is complete independence between the two dimensions. For example, it has been possible to replace most of TFAs with other fatty acids in many foods without modifying significantly the taste of those foods. TFAs were removed because of their adverse impact on health (for an analysis of this case, refer to Unnevehr and Jagmanaitė, 2008). Finally, for other nutrients, an imperfect link might exist. For example, for a given taste, a firm can design products within a range of healthiness. Modifying the taste would imply modifying the range of possible healthiness. In that case, there still exists a link between the two properties, but this link is weaker than the one we consider in this paper. In summary, in the relationship between the taste of a

\(^{12}\)For high level of taxation \( (f \geq f_{\text{max}} = \frac{3t-2-\sqrt{4t^2-14t+28}}{2}) \), firm \( j \) chooses to produce the lowest level of quality \( x_j = 1 \), leading to a strong decrease in the health index.
product and its healthiness, depending on the nutrient, there exists a continuum of cases, ranging from a complete link to a complete independence. In the following, we discuss the possible implications of a weaker link between the taste of the product and its healthiness.

Let us first consider the case of complete independence between the two attributes. Firms have now two levers for action: the product on the horizontal axis and the nutritional quality on the vertical axis. A standard result from the literature is that products are differentiated along one axis and not differentiated along the other axis (Neven and Thisse, 1989). Depending on the weight of the two characteristics in the utility function, two equilibria can emerge. With horizontal dominance, that is when the location matters more than the quality, firms choose maximal differentiation in their location and minimal differentiation in quality. The reverse is true for vertical dominance.\footnote{This result generalizes in the case of $n$ horizontal dimensions. In that case, firms differentiate only along one dimension (Irmen and Thisse, 1998)} Interpreted in the context of our model, with high transportation costs, firms should differentiate on taste but not on healthiness. Conversely, with low transportation costs, firms should differentiate on healthiness but not on taste. However, these results of maximal differentiation on one dimension and minimal differentiation on the other one, emerge in a situation in which there is no cost to modify the variety of the product or the quality of the product. As shown by Olie Lauga and Ofek (2011), in a model with two vertically differentiated attributes and in which quality is costly, firms exploit both dimensions to differentiate their products. In the case of food, it is very likely that making healthier products is costly. Moreover, as exemplified by the case of salt, modifying the taste might also affect the costs of production. Then, even with complete independence between the two attributes, an equilibrium in which firms are differentiated along the two dimensions can emerge when the choice of
the attributes has a cost impact. This means that, in a benchmark case, even when firms have some possibilities to act along the two dimensions, in equilibrium, products can be differentiated both with respect to their taste and to their healthiness.

Interestingly, Duvaleix-Tréguer et al. (2012) proposed a double-differentiation model concerning nutrition and health issues. The goal of this model is somewhat different from ours, as the authors sought to determine how new products that are nutritionally improved can successfully emerge and enter the market in an asymmetrical context in which a low-quality incumbent firm is already in the market. In this paper, the authors assume that the two products have two independent characteristics, one related to taste (horizontal) and another related to health (vertical). Thus, firms have two levers for action: the product variety on the horizontal axis and the nutritional quality on the vertical axis. The authors showed that when consumers have a low WTP for the health attribute, the standard results occur as in Neven and Thisse (1989): Firms choose maximum differentiation on the variety axis and minimum differentiation on the quality axis. However, when at least some consumers have a higher WTP for the health attribute, then the incumbent chooses a higher quality for its product and some differentiation remains on the taste (horizontal) axis. Products are differentiated along the two dimensions. This is another example in which even with independent dimensions differentiation on both attributes can hold.

It is interesting to note that in this model, a move by the entrant firm on the horizontal axis may be accompanied by a move on the vertical axis. For instance, when the quality of the new product increases (vertical axis), the product may shift away from the other product on the variety axis: By doing so, the firm compensates for the increased production cost caused by the quality improvement by lessening the competition intensity on the
horizontal axis. In such a competition setting, firms react not only by adjusting prices and quality, but also by modifying the product variety available on the market and hence the level of substituability between products. This is not possible in our model as the two dimensions are fully dependant and firms have only one lever for action.

Let now consider the case of the MQS policy. In our model, imposing an MQS on the low-quality good induces a change in the location and the quality of the high-quality product. In the model designed by Duvaleix-Tréguer et al. (2012), imposing an MQS is equivalent to an exogenous change in the healthiness of the incumbent product without modifying its taste. Using their framework, it can be shown that, the entrant firm modifies the taste of its product but not the healthiness of the product.\footnote{For analytical results, refer to the Appendix.} In a setting in which taste and healthiness are imperfectly linked, the result would be intermediate. That is, both the location and the healthiness of the high-quality good would change. In case of horizontal dominance, it is likely that the change will be primarily on the horizontal access. Conversely, in the case of vertical dominance, it is likely that the change will occur in primarily on the vertical axis.

The same type of mechanism occurs in the case of a tax on the low-quality good. In our model, the quality of the high-quality good decreases in response to the tax. In the model designed by Duvaleix-Tréguer et al. (2012), the quality of the high quality good is not affected whereas the taste of the product is modified. In a setting in which taste and healthiness are imperfectly linked, the result would be intermediate.

Then, in a context of complete independence between the taste and healthiness of a product, firms are likely to respond to a policy by adjusting one characteristic and leaving
the other unchanged. Our model represents the other extreme case in which the taste and healthiness of the products are perfectly linked. Then, any change in characteristics affects both dimensions. In the more general case of an imperfect link between these two attributes, it is likely that changes induced by a policy would affect both dimensions.

6 Policy Comparison

The three policy interventions analyzed in the previous sections clearly differ in their impacts on market segmentation:

- The MQS policy increases both qualities, increases the demand for the low-quality product and decreases the demand for the high-quality product;

- The tax policy on two products does not change the demands but both qualities increase;

- The tax policy on one product increases the quality of the low-quality product (as long as the tax rate is not too high), decreases the quality of the high-quality product, and increases the demand for the high-quality product.

A common feature of the three policies is to provide incentives to the low-quality firm to increase its product’s healthfulness, that is, to move left. We have shown above that all of the three instruments - MQS, a linear excise tax on the two products, and a linear excise tax on the low-quality product - ameliorate the health index. Nevertheless, depending on the instruments, the effects on the other parameters (quality of the high-quality product, demands, and prices) are not identical. The MQS policy increases the consumer surplus whereas the tax policies decrease it. A linear excise tax on the two products deteriorates
social welfare defined by a populist social planner whereas, as long as the tax rate is not too high, it increases social welfare defined by a paternalistic social planner. Both the MQS and the linear excise tax on one product increase social welfare whether welfare is defined by a populist or a paternalistic social planner.

To go a step further in the analysis and to determine which policy is the most powerful in achieving the regulator’s objective, we now consider the optimal policies that maximize the social welfare, defined by either a populist or a paternalist social planner. Our goal is now to determine which policy is chosen depending on the type of social planner. We only consider the MQS-based and the linear excise tax on the low-quality product policies, and exclude the linear excise tax on the two products as we already know that it is never chosen by the populist social planner. Finally, we compare these optimal policies with information-based policies.

6.1 Optimal policy

We now seek to determine the optimal tax rates and the optimal MQS chosen by the social planners to maximize social welfare and compare the final market outcomes of these optimal policies. The values are numerically computed using Mathematica (see the Appendix for a detailed description of the procedure).

Note that the results depend on the value of the transportation cost $t$ that expresses the strength of consumers preferences for product taste. We have already mentioned (through expression (13)) that we only consider in this paper rather strong values of $t$, and then situations in which it is not easy for consumers to move on the horizontal axis. Tables 2 and 3 show that optimal policies also depend on this parameter. They display the market outcomes of optimal policies chosen by the two social planners for two values of $t : t = 3/4$
and \( t = 9/10 \).

Concerning first the populist social planner (Table 2), we find that:

- When the transportation cost is moderately high \( (t = 3/4) \), the MQS-based policy is preferred to the tax policy, as it allows for higher social welfare. Compared to the tax policy, the variation in consumers’ surplus offsets the firms profit losses and the tax collected. Consumers benefit from a strong average price reduction. The public health output \( H \) is also higher under the MQS. In that case, compared to the benchmark, the market equilibrium is as follows: a smaller market share for the high-quality product (slightly improved) and a larger market share for a strongly improved low-quality product.

- When the transportation cost is high \( (t = 9/10) \), the tax policy is preferred to the MQS-based policy. The market equilibrium is radically different because, compared to the benchmark, it relies on the following: a slight improvement of the low-quality product; a significant decrease in the quality of the high-quality product; and an important move by consumers from the low- (slightly upgraded) to the high-quality product (strongly downgraded), caused by the decrease in the price of the high-quality product. Note that the consumers’ surplus is much lower than under the MQS as the average price is much higher. This is a consequence of the tax which is fully transmitted to the consumer in the context of covered market. In addition, the public health output \( H \) is lower than under the MQS. It is as if, given strong resistance by consumers, the best solution was to not too strongly improve the average quality of the market (like under the MQS), favor consumers switching toward a downgraded high-quality product and maximize the social welfare through tax collection. Note that, in this case, the optimal policy is not the one that maximizes the public health output.
In the case of the paternalist social planner (Table 3), the results change significantly. Indeed, whatever the value of \( t \), the MQS-based policy is preferred. This is because the MQS policy improves significantly the quality of the low-quality product, thus lowering the distortion on this quality while the distortion on the quality of the high quality good slightly increases. The tax policy reduces both distortions but to a small extent, which explains why the welfare increase is lower. Under the MQS, even if many consumers switch from the high- to the low-quality product, the average quality of the market is substantially improved thanks to the large increase in the quality of the low-quality product. In all cases, the public health output increases. Compared to the benchmark and the tax policy, the social welfare is increased thanks to the high increase in the consumers’ surplus (mainly caused by the decrease in the average market price), and despite the drop in the firms’ profits.

Finally, two additional results may be highlighted. First, compared to the benchmark, the tax policy leads to only small variation in the consumer’s surplus. The increase in the social welfare is mainly caused by the amount of the collected tax that offsets the firms’ profit losses (note that this amount could be devoted to information campaigns leading to additional public health benefits).\(^{15}\)

Second, regarding the distributional effects, the two instruments are not identical. To measure these effects, we divide consumers into two groups depending on their health awareness. Consumers characterized by \( 1/2 < \lambda \leq 1 \) are health-conscious consumers, and consumers characterized by \( 0 \leq \lambda \leq 1/2 \) are non-health-conscious consumers. In the case of a populist social planner, the consumer surplus of non-health-conscious consumers is

\(^{15}\)Note that, in the case of elastic demand, if the market were not covered, the change in welfare due to the tax would be smaller, as an increase in the price would cause deadweight losses which is not the case in our setting in which the market is covered.
Table 2: Comparison of optimal policies designed by a populist social planner

always lower under the tax policy than in the benchmark case, whatever the transportation cost. The tax policy is then clearly regressive in this case, whereas the MQS-based policy increases the surpluses of both consumer types. The reason is that the MQS leads to a strong increase in the quality of the low-quality product that is cheaper (and even cheaper than in the benchmark, as the competition intensity increases). In the case of the paternalist social planer, the two policies are progressive, as they increase both consumers surpluses and more strongly those of non-health-conscious consumers.

6.2 Information policy versus policies changing the market environment

Thus far, we have considered policies targeting changes in the market environment facing consumers. The goal is to favor health benefits even for non-informed and non-health-conscient consumers.
\[
\begin{array}{cccccccc}
& t = 3/4 & & t = 9/10 & \\
& BM & Tax & MQS & BM & Tax & MQS & \\
& f = xx & \bar{x} = 0.381 & f = xxx & \bar{x} = 0.418 & \\
x_i & 0.050 & 0.200 & 0.000 & 0.018 & 0.232 & 0.000 & \\
x_j & 0.950 & 0.908 & 0.381 & 0.982 & 0.938 & 0.418 & \\
p_i & 1.127 & 1.028 & 0.689 & 1.350 & 1.212 & 0.763 & \\
p_j & 0.677 & 0.794 & 0.578 & 0.868 & 1.034 & 0.659 & \\
d_i & 0.500 & 0.667 & 0.329 & 0.500 & 0.722 & 0.349 & \\
d_j & 0.500 & 0.333 & 0.671 & 0.500 & 0.278 & 0.651 & \\
\bar{p} & 0.901 & 0.950 & 0.614 & 1.109 & 1.163 & 0.695 & \\
\pi_i & 0.338 & 0.472 & 0.062 & 0.434 & 0.663 & 0.092 & \\
\pi_j & 0.338 & 0.118 & 0.259 & 0.434 & 0.098 & 0.319 & \\
H & 0.500 & 0.564 & 0.744 & 0.500 & 0.572 & 0.728 & \\
\pi_i + \pi_j & 0.675 & 0.590 & 0.321 & 0.868 & 0.761 & 0.411 & \\
CS & 1.528 & 1.563 & 2.051 & 1.301 & 1.351 & 1.951 & \\
CSh & 0.727 & 0.752 & 1.010 & 0.617 & 0.651 & 0.961 & \\
CSi & 0.802 & 0.811 & 1.041 & 0.684 & 0.7000 & 0.990 & \\
SW2 & 2.203 & 2.298 & 2.373 & 2.169 & 2.300 & 2.361 & \\
Tax & - & 0.145 & - & - & 0.189 & - & \\
\end{array}
\]

v=2; BM stands for benchmark; Tax stands for tax on the bad product

In this table, consumer surplus is evaluated from the paternalistic point of view, that is assuming \( \lambda = 1 \)

Table 3: Comparison of optimal policies designed by a paternalistic social planner
conscious consumers. In practice, however, public planners often develop information policies. In the following, we analyze whether market environment policies perform better, in terms of health and welfare impacts, than policies exclusively based on education and information campaigns.

To determine to what extent information-based policies may be more or less effective than policies changing the market environment, we construct the following scenarios. First, we consider a situation in which no consumer is aware of the health impact of nutrient intake \( (\lambda = 0) \) and the transportation cost is high \( (t = 0.9) \). From this initial starting point, policy makers may decide to implement an MQS-based policy (if the planner is paternalist) or a tax policy (if the planner is populist) without seeking for changes in consumer perceptions of the health impacts of food intakes. On the contrary, policy makers can decide to implement information campaigns to make consumers more aware of the health impacts of food intakes. In this latter case, we assume that all consumers become fully aware \( (\lambda = 1) \) thanks to a perfectly efficient information campaign (we do not take into account the cost of this information campaign). By definition, in this case, the social welfare is the same for the populist and paternalist social planners.

Table 4 displays public health outcomes and social welfare. Compared to the initial starting point, both the tax policy and the MQS policy improve welfare and health outcomes. However for the paternalist social planner who chooses the MQS policy, social welfare and health index increase considerably more under this policy than under an information policy. In fact, the quality distortions in equilibrium, discussed in Section 5, still hold with perfectly informed consumers: The quality of the high-quality product is too high, and the quality of the low-quality product is too low. In other words, under an
Table 4: Comparison of the impact of market policies versus information policy

<table>
<thead>
<tr>
<th>Policy Type</th>
<th>Health Index</th>
<th>Welfare (SW1)</th>
<th>MQS: ( \bar{x} )</th>
<th>Tax Policy: ( f = 0.770 )</th>
<th>MQS: ( \bar{x} = 0.821 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Populist Social Planner</td>
<td>0.381</td>
<td>1.776</td>
<td>0.461</td>
<td>0.619</td>
<td></td>
</tr>
<tr>
<td>No Policy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax</td>
<td>0.385</td>
<td>1.852</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paternalistic Social Planner</td>
<td>0.381</td>
<td>2.157</td>
<td>0.683</td>
<td>0.619</td>
<td></td>
</tr>
<tr>
<td>No Policy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax</td>
<td>0.423</td>
<td>2.264</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \lambda = 0 \) and \( \lambda = 1 \) with \( t = 0.9 \), \( v = 2 \); MQS and tax are chosen to maximize the related welfare.

In the case of the populist social planner, it turns out that an efficient information campaign allows for a higher health index and social welfare than the tax policy (with unconscious consumers). The choice between the two then depends on the cost of an information campaign. If it is not too expensive to make all consumers aware of health dimensions, the information policy must be prioritized. If it is costly, the tax policy has to be chosen.

7 Conclusion

In this article, we propose an original model of product differentiation intended to estimate to what extent tax- and standard-based policies can contribute to improving the nutritional quality of foods and, in turn, the health status of the population. On the supply side, a single characteristic, that is, the content of the product in some ‘unhealthy’ nutrient is used to define the product. On the demand side, this characteristic affects the consumers utility in two ways: first through its impact on product taste, and second through its impact on health. The main contribution of our analysis is to evaluate, in this setting, market impacts
of alternative policies, integrating strategic reactions of firms both in terms of prices and product characteristics. In addition we estimate social welfare impacts as defined by either a populist or a paternalist social planner. A populist social planner evaluates the consumer surplus on the basis of the utility function of consumers, as he does not fully incorporate into the consumer surplus the ‘true’ health impact of the bad nutrient intakes. Alternatively, a paternalistic social planner fully integrates the health impact of consumption.

In the absence of any policy intervention, we find that in equilibrium (i) products are more differentiated than in the standard horizontal differentiation model - this is due to the consumers’ heterogeneity in both health and taste dimensions; and (ii) for a welfare-maximizing planner, the two products are too differentiated- the quality of the high-quality product is too high, and the quality of the low-quality product is too low. This distortion may justify public policy interventions intended to influence quality decisions made by firms to restore an optimal market segmentation.

Among the three policy options analyzed, the MQS policy and the taxation of the low-quality product are the most preferred by a social planner. Under an MQS, both firms improve the quality of the products and product differentiation decreases, as it is more costly for the high-quality firm to improve the quality of its product. This policy improves the health outcome and is welfare increasing whether welfare is evaluated by a populist or a paternalist social planner. Taxation of the low-quality product also improves health and welfare. This policy induces a change in the quality of products, which is consistent with the social planner’s perspective. However, this policy leads to an increase in the average price of the product whereas the MQS policy decreases the average price.

The definition of an optimal policy depends on the consumers’ willingness to accept
taste modifications and the type of social planner. If the transportation cost is moderately high, the MQS policy is chosen by the two social planners. If the transportation cost is very high, the populist social planner chooses to tax the low-quality product whereas the paternalist social planner still prefers the MQS-based policy.

Information policies improving the consumers’ perception of health impacts of bad nutrient intakes may lead to better public health outcomes than the benchmark. However, for a paternalist social planner, an MQS-based policy always allows for higher levels of welfare (and the health index) than an information policy alone. The main reason is that, even with consumers who are perfectly informed of the health impacts of food choices, market segmentation is not optimal and quality distortions still prevail. For a populist social planner, the results are mixed and depend on the cost of the information policy. If the information policy is not too expensive, this policy must be prioritized; otherwise, the tax policy performs better than the information policy alone.

Overall, our results have two important consequences for nutritional policy design. Whereas some policy makers are sometimes reluctant to go beyond consumer information and education interventions, our analysis suggests that policies targeting changes in food quality must always complement information policies. A large part of the public debates related to nutrition and health policies has focused on fiscal interventions (fat tax, taxes on soft beverages). Our analysis does not suggest prioritizing such instruments, except in the case of a populist social planner in presence of strong taste resistances among consumers. Even in this case, note that consumer surplus does not increase significantly. In fact, the tax policy is regressive, thus lowering the surplus of the less-health-conscious consumers while increasing the surplus of the more-health-conscious consumers. Moreover, we also show that
if the tax rate is too high, firms’ strategic reactions may lead to unintended consequences. Thus, an excessively high tax rate leads the low-quality firm to degrade the quality of its product, which leads to a strong negative impact on the health outcome of the policy. It should be stressed that in all other cases, standard-based policies should be prioritized. Practical difficulties of such policies that could however limit their implementation would have to be analyzed in greater detail.

These results must be considered while taking into account some limitations of our model. An important assumption of the model was the market coverage assumption. Thus, we assumed that the global demand is fixed. The demand for each firm is elastic, as consumers switch from one product to another, but the overall demand is inelastic. In practice, this means that we implicitly consider markets for products that are consumed by all consumers, which are thus difficult to substitute in the diet. The limit is mainly related to the impact of an increase in the quality of the ‘bad’ product, which is the cheaper product. The tax policies analyzed above lead to an increase in the price of the low-quality good. We assume here that this does not discourage consumption as every consumer continues to buy one of the products on the market. In practice, it is possible that some consumers will cease consuming this product. This would affect their surplus, but this would also affect the profit of the firm selling that product, thus limiting the positive impact of the policy. Conversely, in the case of the MQS policy, its interest could be larger in a context of elastic demand, as this policy leads to a decrease in the price of the low-quality good. A second limitation of the model is related to the assumption that a consumer buys one unit of a product. By doing so, we do not capture an individual quantity effect that might occur when the characteristic of the product changes. It is
possible that a decrease in the content of a ‘bad’ nutrient generates an increase in the quantity consumed, as consumers might believe that they ‘can’ eat more of the product because it is now safer. Addressing this second issue requires a radical change in the model to endogeneize quantity decisions by consumers.

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References


49


8 Appendix

All results are obtained using Mathematica.

8.1 Determination of the Perfect Nash equilibrium

8.1.1 Demand functions and associated conditions

Given the indifference line (2), we define demands as functions of \( p_i, p_j, x_i, x_j \). We define \( x_0 \) the location of the indifferent consumer with \( \lambda = 0 \) and \( x_1 \) the location of the indifferent consumer with \( \lambda = 1 \). We have \( x_1 - x_0 = 1/2t \leq 1 \) (as \( t \geq 1/2 \)). Depending on the value of \( x_0 \) and \( x_1 \), demand is written as:

- Case 1: \( x_0 \geq 0 \) and \( x_1 \leq 1 \): This is the standard case and we have
  \[
  D_1^i = \frac{1}{4t}[t(x_i + x_j) + \frac{p_j - p_i}{x_j - x_i} + \frac{1}{2}]
  \]
- Case 2: \( x_0 \leq 0 \) and \( x_1 \leq 1 \): Demand for product \( i \) is reduced to consumers with a 'small' \( x \) and a 'high' \( \lambda \). We have
  \[
  D_2^i = \frac{1}{4t}[1 + t(x_i + x_j) + \frac{p_j - p_i}{x_j - x_i}]^2
  \]
- Case 3: \( 0 \leq x_0 \leq 1 \) and \( x_1 \geq 1 \): In this case, demand for product \( j \) is reduced to consumers with a 'high' \( x \) and a 'low' \( \lambda \). We have
  \[
  D_3^j = \frac{1}{4t}[2 - t(x_i + x_j) - \frac{p_j - p_i}{x_j - x_i}]^2
  \]

We also use \( D_i + D_j = 1 \) to define the demand of the other product, and we exclude cases in which \( D_i = 0 \) or \( D_j = 0 \).

For each of the three cases defined above, we determine the Nash price equilibrium in the second stage of the game, that is given the characteristics \( x_i \) and \( x_j \). We denote \( p_l^i \), \( l \in \{1, 2, 3\} \) the price equilibrium for product \( i \) in case \( l \). In case 1, we find that:

\[
 p_i(p_j) = \frac{p_j}{2} + \frac{1}{4}[1 - 3x_i + x_i^2 + x_j] + \frac{t}{2}[x_j^2 - x_i^2]p_j(p_i) = \frac{p_i}{2} + \frac{1}{4}[1 - 3x_j + x_j^2 + x_i] + \frac{t}{2}[2(x_j - x_i) - (x_j^2 - x_i^2)]
\]

from which we deduce the prices at the second-stage equilibrium (\( p_l^i \) is given in (9) and \( p_l^j \) is given in (10). Note that both reaction functions are upward sloping.

Using the expressions of the equilibrium prices at the second stage, we rewrite the indifferent consumer line, which is now a function of \( t, x_i \) and \( x_j \) and deduce the following conditions on \( x_i \) and \( x_j \):

- Case 1: \( \frac{4(1-t)}{1+2t} \leq x_i + x_j \leq \frac{2(4t-1)}{1+2t} \)
• Case 2: \(0 \leq x_i + x_j \leq \frac{4(1-t)}{1+2t}\)

• Case 3: \(\frac{2(4t-1)}{1+2t} \leq x_i + x_j \leq 2\)

In the following, we will use these conditions as well as \(0 \leq x_i \leq x_j \leq 1\). Given the specification of the model, firm \(i\) produces the high quality, firm \(j\) produces the low quality and qualities are in the range \([0,1]\).

### 8.1.2 Existence of an equilibrium

Solving analytically the model in ‘case 1’ demand, leads to the best reply functions (in characteristic, that is in location) which are given by:

\[
x_i(x_j) = \frac{x_j}{3} + \frac{1 - 4t}{3(1+2t)}x_j(x_i) = \frac{x_j}{3} + \frac{1 + 8t}{3(1+2t)}
\]

from which we deduce the equilibrium locations given by (11) and (12). Let’s denote the profit of firm \(i\) for this potential equilibrium \(\pi_i^B\). To prove that the pair of locations defined by (11) and (12) is chosen at the perfect Nash equilibrium, we have to prove that \(x_i^B\) is the best reply in quality to \(x_j^B\), \(\forall x_i \leq x_j^B\) and that \(x_j^B\) is the best reply in quality to \(x_i^B\), \(\forall x_j \geq x_i^B\).

We now present, the method used to analyse if \(x_i^B\) is the best reply in quality to \(x_j^B\). First, given the way we determined \(x_i^B\) and \(x_j^B\), \(x_i^B\) is the best reply to \(x_j^B\) as long as the demands are defined by \((D_i^1, D_j^1)\). Second, let suppose we are in ‘case 2’ demand. Given \(x_j^B = \frac{1 + 5t}{2(1+2t)}\), case 2 is possible only if \(t \leq 7/13\) as \(x_j \geq \frac{4(1-t)}{1+2t}\) for \(t \geq 7/13\). Finally, it is easy to check that when \(x_i\) is sufficiently large, we face case 3 for demand. Thus, for \(t \in [1/2, 7/13]\) we have to determine if \(x_i^B\) is the best reply to \(x_j^B\) with demands defined by case 2 or 3 (obviously depending on the value of \(x_i\)). For \(t \in [7/13, 1]\) we have to determine if \(x_i^B\) is the best reply to \(x_j^B\) with demands defined by ‘case 3’ only. A similar analysis is developed to analyse if \(x_j^B\) is the best reply to \(x_i^B\) (given \(x_i^B\), it also happens that both ‘case 2’ and ‘case 3’ demand need to be explored when \(t \leq 7/13\) and only ‘case 2’ demand when \(t \geq 7/13\)). In the following we detail the method used in one situation. Then, we sum-up the results found for the other cases, using a similar method.
Assume $t \geq 7/13$, we have to determine the profit of producer $i$, when the demand is defined by $(D_i^B, D_j^B)$. Prices at the second stage are $p_i^3$ and $p_j^3$. $x_j^B$ is the best reply to $x_j^B$ iff:

$$\pi_i[p_i^3(x_i, x_j^B), p_j^3(x_i, x_j^B)] \leq \pi_i^B, \quad \forall x_i \in \left[\frac{2(4t-1)}{(1+2t)} - x_j^B, x_j^B\right]$$

(22)

The restriction on $x_i$ comes from the conditions on the demand in case 3. We use Mathematica to test if (22) is true. We determine $p_i^3(x_i, x_j^B)$ and $p_j^3(x_i, x_j^B)$ in function of $t$ from which we determine $\pi_i[p_i^3(x_i, x_j^B), p_j^3(x_i, x_j^B)]$ which is now written $\pi_i^3(x_i, x_j^B)$. Denoting $\bar{x}_i$ the lower value of $x_i$, that is $\frac{2(4t-1)}{(1+2t)} - x_j^B = \frac{11t-5}{2(1+2t)}$. We first determine $\pi_i^3(\bar{x}_i, x_j^B) - \pi_i^B$ and find (numerically) that it is negative $\forall t \in [7/13, 1]$. Then we determine $\frac{\partial \pi_i^3(x_i, x_j^B)}{\partial x_i}$ and find (numerically) that it is negative $\forall x_i \in [\bar{x}_i, x_j^B]$. Thus, $\pi_i^3(x_i, x_j^B)$ decreases with $x_i$ (note that when $x_i = x_j^B$ then the profit of firm $i$ is 0). This proves that (22) is true $\forall t \in [7/13, 1]$ meaning that $x_i^B$ is the best reply in quality to $x_j^B$. A similar analysis leads to the conclusion that $x_j^B$ is the best reply in quality to $x_i^B$. We now sum-up the results of the different cases:

- $t \in [7/13, 1]$: the couple of quality $x_i^B$, $x_j^B$ is a perfect Nash equilibrium.
- $t \in [1/2, 7/13]$
  - $t \in [1/2, 0.524766]$ : the couple of quality $x_i^B$, $x_j^B$ is not an equilibrium. Thus in this range we find that $x_i = 0$ is the best reply to $x_j^B$. We also find that $x_j = 1$ is the best reply to $x_i^B$. The value of the bound is solved numerically.
  - $t \in [0.524766, 7/13]$ : the couple of quality $x_i^B$, $x_j^B$ is an equilibrium.

8.1.3 Unicity of the equilibrium

For $t > 0.524766$, we proved that $x_i^B$, $x_j^B$ constitutes a perfect Nash equilibrium. We now have to determine if there are other equilibria. Let us consider the ‘case 2’ demand. This corresponds to $x_i + x_j \leq \frac{4(1-t)}{1+2t}$. We first determine the second stage price equilibrium $p_i^2(x_i, x_j)$ and $p_j^2(x_i, x_j)$. As it is not possible to find analytically the optimal qualities, we first look for the best reply of producer $i$ to a given $x_j$. We get 3 possible candidates. It turns out that none of these candidates are in the range $[0, x_j]$. However, for a given $x_j$,
the profit of firm $i$ decreases with $x_i$ when $x_i \in [0, x_j]$. We thus find that the best reply of producer $i$ is $x_i^2(x_j) = 0, \forall t \in [1/2, 1]$. We now have to determine the best reply of producer $j$ to $x_i = 0$, that is $x_j^2(0)$. To do so, we compute the profit of firm $j$, given the price at the second stage, with $x_i = 0$ and maximizes it wrt $x_j$. We get 7 solutions. It turns out that only one solution is in the range $[0, 1]$. We also checked the conditions on the demand ($x_i + x_j = x_j \leq \frac{4(1-t)}{1+2t}$). We find that there exists a solution only for $t \in [\frac{1}{2}, \frac{\sqrt{33}-1}{8} \simeq 0.593]$.

The couple of qualities $\{0, x_j^2(0)\}$ is an equilibrium, if the profit producer $j$ gets when playing $x_j^2(0)$ is larger than what he would get by playing $x_j^1(0)$, defining it as his best response assuming the demand is defined by ‘case 1’ (note that we do not have to explore the ‘case 3’ demand as this case is not compatible with $x_i = 0$). If the profit of producer $j$ when playing $x_j^1(0)$ is greater than the one he gets by playing $x_j^2(0)$, then the couple of qualities $\{0, x_j^2(0)\}$ is not an equilibrium as the best reply of producer $j$ to $x_i = 0$ is in the ‘case 1’ demand (and in that case $x_i$ is no longer the best reply to $x_j$). Applying the above analysis, we find that there exists an equilibrium in the ‘case 2’ demand when $t < 0.58934$. (we are only able to solve it numerically). We develop the same analysis in the ‘case 3’ demand. We find a similar result that there exists an equilibrium with $x_j = 1$ under the same conditions. To sum up:

- $t \in [1/2, 0.524766]$ We have two equilibria: an equilibrium in ‘case 2’ demand with $x_i = 0$, and an equilibrium in ‘case 3’ demand with $x_j = 1$
- $t \in [0.524766, 0.58934]$ We have three equilibria: $\{x_i^B, x_j^B\}$, and the two equilibria defined above.
- $t \in [0.58934, 1]$ We have a unique perfect Nash equilibrium $\{x_i^B, x_j^B\}$

### 8.2 Characterization of the Perfect Nash Equilibrium ($t \in [\tilde{t}, 1]$)

#### 8.2.1 Equilibrium prices, profits, consumer surplus and social welfare

Substituting $x_i^B$, $x_j^B$ into the second stage prices (Eq. (11) and (12)), we get the equilibrium prices:

$$p_i^B = \frac{1 + 10t + 49t^2 + 48t^3}{8(1+2t)^2}, \quad (23)$$
\[ p^B_j = \frac{1 - 2t + 25t^2 + 48t^3}{8(1 + 2t)^2}. \]  

(24)

To get the demand at equilibrium, substitute (23), (24) and \( x^B_i, x^B_j \) into (3), (4), we get:

\[ D_i = D_j = \frac{1}{2}. \]  

(25)

From which we deduce:

\[ \Pi_i = \Pi_j = \frac{3t^2}{2(1 + 2t)}. \]  

(26)

The distance between the two firms provides a measure of the product differentiation. We have \( x^B_j - x^B_i = \frac{3t}{1 + 2t} \). The health index is computed by substituting (11), (12), and (25) into (7): \( h_{ij} = \frac{1}{2} \)

### 8.2.2 Consumer surplus

Consumer surplus is written as:

\[
CS = \int_0^{x_0} \int_0^1 U_i d\lambda dx + \int_{x_0}^{x_1} \int_0^1 U_i d\lambda dx + \int_{x_0}^{x_1} \int_0^1 U_j d\lambda dx + \int_0^{x_0} \int_0^1 U_j d\lambda dx, 
\]

(27)

with the indifference curve given by: \( \lambda(x) = 2tx - t(x_j + x_i) - \frac{p^B_j - p^B_i}{x_j - x_i} \). By substituting the equilibrium values of \( x^B_i, x^B_j, p^B_i \) and \( p^B_j \), we get:

\[
CS = \frac{9 + 8t - 170t^2}{48(1 + 2t)}. 
\]

(28)

### 8.2.3 Social welfare

Welfare, for a populist social planner, is defined as \( SW = \Pi_i + \Pi_j + CS \). From which, we deduce:

\[
SW1 = \frac{9 + 8t - 26t^2}{48(1 + 2t)}. 
\]

(29)

Welfare, for a paternalistic social planner, is defined as :

\[
SW2 = \Pi_i + \Pi_j + \int_0^{x_0} \int_0^1 U'_i d\lambda dx + \int_{x_0}^{x_1} \int_0^1 U'_i d\lambda dx + \int_{x_0}^{x_1} \int_0^1 U'_j d\lambda dx + \int_0^{x_1} \int_0^1 U'_j d\lambda dx, 
\]

(30)
By substituting the equilibrium values of $x_i^B$, $x_j^B$, $p_i^B$ and $p_j^B$, we get:

$$SW2 = \frac{15 + 32t - 26t^2}{48(1 + 2t)}$$  \hspace{1cm} (31)

### 8.2.4 Qualities are strategic complements

As discussed above, the best reply functions in characteristic are given by:

$$x_i(x_j) = \frac{x_j}{3} + \frac{1 - 4t}{3(1 + 2t)} x_j(x_i) = \frac{x_i}{3} + \frac{1 + 8t}{3(1 + 2t)}$$

Both best reply functions are upward sloping, meaning that the locations $x_i$ and $x_j$ are strategic complements.

### 8.3 Optimal qualities for the social planner

Given (14), we have

$$SW1 = \frac{x_j - x_i + 3t(2 - 2x_j + x_j^2 - x_i^2) + 4t^2(3x0(x_j - x_i)(1 - x_0) - 1 - 3x_j^2)}{12t} - \frac{1 - x_j^2}{2(x0 + 1/4t)} - \frac{1 - x_i^2}{2(1 - 1/4t)}$$  \hspace{1cm} (32)

Maximising this function wrt $x_0$ leads to the optimal $x_0 = \frac{(x_i + x_j)(1 + 2t) - 2}{4t}$. Using the optimal $x_0$, we then maximise the welfare function wrt $x_i$ and $x_j$ leading to the optimal qualities.

We use the same methodology to determine the optimal qualities in the case of the paternalistic social planner.

### 8.4 Instrument: MQS

Stage 2 of the game is not modified and thus (9), (10) apply. $x_i^S$ is found by maximising $\Pi_i(p_i^*(x_i, \bar{x}), p_j^*(x_i, \bar{x}))$ over $x_i$. The optimal solution is given by (19). Following the same method as for the benchmark, we deduce prices, demands, profits, health index, consumer surplus and welfare at the equilibrium. They are obviously function of $\bar{x}$ which is exogenously set. We provide in the following the analytical expressions of all those variables.
8.4.1 Equilibrium prices

\[ p_M^i = \frac{[27 + 18(\bar{x} - 4)(1 + 2t) + [59 - 38\bar{x} + 11\bar{x}^2 + 16t(1 + \bar{x}^2)](1 + 2t)^2]}{54(1 + 2t)^2} \]  
(33)

\[ p_M^j = \frac{[36 + (1 + 2t)[-11 - 46\bar{x} + 19\bar{x}^2 + 8t(7 - 2\bar{x})(1 + \bar{x})]]}{54(1 + 2t)} \]  
(34)

and

\[ \frac{\partial p_M^i}{\partial \bar{x}} \bigg|_{x_B^j} = \frac{16t^2 + (3t - 1)}{6(1 + 2t)} > 0 \]

\[ \frac{\partial p_M^j}{\partial \bar{x}} \bigg|_{x_B^j} = \frac{(t - 1)}{2(1 + 2t)} < 0 \]

8.4.2 Demands

\[ D_M^i = \frac{-1 + 4t + 2\bar{x}(1 + 2t)}{18t} \]  
(35)

\[ D_M^j = \frac{1 + 14t - 2\bar{x}(1 + 2t)}{18t} \]  
(36)

and

\[ \frac{\partial D_M^i}{\partial \bar{x}} \bigg|_{x_B^j} = \frac{1 + 2t}{9t} > 0 \]

\[ \frac{\partial D_M^j}{\partial \bar{x}} \bigg|_{x_B^j} = -\frac{1 + 2t}{9t} < 0 \]

8.4.3 Profits

\[ \Pi_M^i = \frac{[2\bar{x} - 1 + 4t(\bar{x} + 1)]^3}{486t(1 + 2t)} \]  
(37)

\[ \Pi_M^j = \frac{(1 + 14t - \bar{x}(4t + 2))^2(-1 + 4t + \bar{x}(2 + 4t))}{486t(1 + 2t)} \]  
(38)

and

\[ \frac{\partial \Pi_M^i}{\partial \bar{x}} \bigg|_{x_B^j} = t > 0 \]

\[ \frac{\partial \Pi_M^j}{\partial \bar{x}} \bigg|_{x_B^j} = -\frac{t}{3} < 0 \]
8.4.4 Health index

\[ h_{ij}^M = \frac{1 + 46t + 124t^2 + 4\bar{x}^2(1 + 2t)^2 - 2\bar{x}(2 + 23t + 38t^2)}{54t(1 + 2t)} \]  

(39)

and

\[ \frac{\partial h_{ij}^M}{\partial \bar{x}} \bigg|_{x_j^B} = -\frac{1}{3} < 0 \]

8.4.5 Consumer surplus

\[ CS^M = \frac{1}{3888t(1 + 2t)}(-31 - 24\bar{x}^2(2 + 49t)(1 + 2t)^2 + 32\bar{x}^3(1 + 2t)^3 + \]

\[ 6\bar{x}(1 + 2t)[13 + 4t(49 + 58t)] - 4t[-75 + 4t(3 + 650t)] \]  

(40)

and

\[ \frac{\partial CS^M}{\partial \bar{x}} \bigg|_{x_j^B} = \frac{1}{72t} - t < 0 \]

8.4.6 Social welfare

\[ SW^1_M = \frac{1}{3888t(1 + 2t)}(-47 - 3616\bar{x}^3 - 120\bar{x}^2(2 + 13t)(1 + 2t)^2 + 160\bar{x}^3(1 + 2t)^3 + \]

\[ 6\bar{x}(29 + 318t + 1104t^2 + 1168t^3) - 1104t^2 + 204t) \]  

(41)

and

\[ \frac{\partial SW^1_M}{\partial \bar{x}} \bigg|_{x_j^B} = \frac{1}{72t} - \frac{t}{3} < 0 \]

\[ SW^2_M = \frac{1}{3888t(1 + 2t)}(-47 - 3616\bar{x}^3 - 24\bar{x}^2(4 + 65t)(1 + 2t)^2 + 160\bar{x}^3(1 + 2t)^3 + \]

\[ 6\bar{x}(-13 + 6t + 648t^2 + 1168t^3) + 3360t^2 + 1644t) \]  

(42)

and

\[ \frac{\partial SW^2_M}{\partial \bar{x}} \bigg|_{x_j^B} = -\frac{1}{6} - \frac{1}{72t} - \frac{t}{3} < 0 \]

8.4.7 Optimal MQS

We first study the case of the populist social planner. We have proved that \( \frac{\partial SW^1_M}{\partial \bar{x}} \bigg|_{x_j^B} < 0 \). Then there exists a restrictive MQS which improves welfare. To determine the optimal MQS, we need to check some conditions when optimizing the welfare function over \( \bar{x} \). Thus, three constraints need to hold: \( x_i \geq 0, \ x_0 \geq 0, \) and \( x_1 \leq 1 \). The second and third constraint ensure that demands are defined by ‘case 1’. Given the prices and the qualities.
at the equilibrium, the first condition is equivalent to $\bar{x} \geq \frac{4t-1}{1+2t}$, the second condition is equivalent to $\bar{x} \geq \frac{11-8t}{4(1+2t)}$, and the third condition is always satisfied. The first two constraints are identical for $t = 5/8$.

Let first consider the case, $t \leq 5/8$, then condition $x_0 \geq 0$ restricts the existence of an equilibrium to $\bar{x} \in [\frac{11-8t}{4(1+2t)}, x_j^B]$ and $x_i^M$ is given by (19). It is easy to show that $SW_1^M$ decreases over $[\frac{11-8t}{4(1+2t)}, x_j^B]$. Then, the optimal MQS when demand is defined by case 1, is given by $\bar{x} = \frac{11-8t}{4(1+2t)}$. We then deduce the value of the surplus at the optimal MQS, conditional on being in ‘case 1’ demand. Thus, it is possible that an equilibrium exists in ‘case 2’ demand. This is possible as in the present case (as opposed to the benchmark case), the quality of product $j$ is bounded. Using the expression of ‘case 2’ demand as well as the associated second stage price equilibrium, we found that an equilibrium with $x_i = 0$ exists. That is to say that the MQS can be defined for lower values than $\frac{11-8t}{4(1+2t)}$. We then determine the welfare in this case and compare it to the one obtained previously. It happens that there exists a more restrictive MQS which leads to a higher welfare, meaning that the optimal MQS is defined in ‘case 2’ demand.

Let now consider the case, $t \geq 5/8$. The condition on $x_i \geq 0$ is the most restrictive and $x_i^M$ is given by (19) as long as $\bar{x} \geq \frac{4t-1}{1+2t}$. Otherwise, we have $x_i^M = 0$ and we need to check if a type 2 condition ($x_0 \geq 0$) is satisfied (given $x_i^M = 0$): we have $x_0 \geq 0 \Leftrightarrow \bar{x} \geq \frac{4(1-t)}{1+2t}$. Let’s now determine the sign of $\frac{\partial SW_1^M}{\partial \bar{x}} \big|_{\bar{x} = \frac{4(1-t)}{1+2t}}$. We find that it is negative for all $t \in [5/8, 1]$.

We also find that $\frac{\partial SW_1^M}{\partial \bar{x}} < 0 \forall \bar{x} \in \left[\frac{4(1-t)}{1+2t}, x_j^B\right]$. Then, the optimal MQS is determined by $\frac{\partial SW_1^M(0,x)}{\partial \bar{x}} = 0$. We get $MQS^* = \frac{2(5+38t+56t^2-\sqrt{(1+2t)^2(-5+70t+304t^2)})}{15(1+2t)^2}$. This is an ‘interior’ solution ($\bar{x} > \frac{4(1-t)}{1+2t}$) as long as $t \geq 33+\sqrt{137} \simeq 0.6574$.

When $t$ is lower than this threshold, then the optimal MQS, providing the demand is defined by ‘case 1’, is given by $\frac{4(1-t)}{1+2t}$. Finally, we check (numerically) if a more restrictive MQS leads to higher surplus considering the case 2 demand. Focusing on two specific cases ($t = 3/4$ and $t = 9/10$), we find that in both cases the optimal MQS is defined for the case 1 demand.

We use the same methodology to find the optimal MQS defined by a paternalistic social
planner. We focus on $t \geq 5/8$. As for the populist social planner, we find that the optimal MQS is determined by 
$$\frac{\partial SW}{\partial x}(M(0, x)) = 0.$$ 
We get 
$$MQS^* = \frac{2(2+32t+56t^2-\sqrt{(1+2t)^2(19+82t+304t^2)})}{(15(1+2t)^2)}.$$ 
This is an ‘interior’ solution ($\bar{x} > \frac{4(1-t)}{1+2t}$) as long as $t \geq \frac{37+\sqrt{213}}{68} \simeq 0.758743$. When $t$ is lower than this threshold, then the optimal MQS, providing the demand is defined by ‘case 1’, is given by 
$$\frac{4(1-t)}{1+2t}.$$ 
Finally, we check (numerically) if a more restrictive MQS leads to higher surplus considering the ‘case 2’ demand. Focusing on two specific cases ($t = 3/4$ and $t = 9/10$), we find that for $t = 3/4$, the optimal MQS is defined for the ‘case 2’ demand whereas for $t = 9/10$ the optimal MQS is defined for the ‘case 1’ demand.

8.5 Linear Excise Tax - Scenario 1: Taxing both products

The analysis is similar to the analysis of the benchmark case. To determine the equilibrium, we use the new profit functions defined by (20) and (21).

8.5.1 Optimal qualities

$$x_i^f = \frac{1-2f-t}{2(1+2t)} < x_i^B$$
$$x_j^f = \frac{1-2f+5t}{2(1+2t)} < x_j^B$$

We also deduce that

$$x_j^f - x_i^f = \frac{3t}{1+2t} = x_j^B - x_i^B$$

8.5.2 Equilibrium prices

$$p_i^f = p_i^B + \frac{f(8t-f-2t-4tf-2t^2+2)}{2(1+2t)^2}$$
$$p_j^f = p_j^B + \frac{f(8t-f-2t-4tf+5t^2+2)}{2(1+2t)^2}$$

We have $p_i^f - p_i^B > 0$, $p_j^f - p_j^B > 0$ and $p_j^f - p_j^B > p_i^f - p_i^B$.

8.5.3 Demands and Profits

$$D_i^f = D_j^f = \frac{1}{2} = D_i^B = D_j^B$$
It is easy to check that the line defining indifferent consumers is \( \lambda = \frac{1}{2} + t(2x - 1) \) which is identical to that in the benchmark case. Note that the consumer \( (\frac{1}{2}, \frac{1}{2}) \) belongs to this line.

\[
\Pi_i^f = \Pi_j^f = \frac{3t^2}{2(1 + 2t)} = \Pi_i^B = \Pi_j^B
\]

8.5.4 Health index

\[
h_{ij}^f = \frac{1 + 2f + 2t}{2(1 + 2t)} > h_{ij}^B
\]

8.5.5 Consumer surplus

\[
CS_i^f = \frac{9 + 8t - 170t^2 - 24f(1 + 2t - f)}{48(1 + 2t)} = CS_i^B - \frac{24f(1 + 2t - f)}{48(1 + 2t)} < CS_i^B
\]

8.5.6 Welfare

\[
SW_1^f = -\frac{-9 + 24f^2 - 8t + 26t^2}{48(1 + 2t)} < SW_1^B
\]

\[
SW_2^f = \frac{15 + 24f - 24f^2 + 32t - 26t^2}{48(1 + 2t)}
\]

and

\[
SW_2^f - SW_2^B = \frac{(1 - f)f}{2(1 + 2t)} > 0 \text{ if } f < 1
\]

8.6 Linear Excise Tax - Scenario 2: Taxing only product \( j \)

The analysis of this case leads to much more complex expressions.\(^{16}\) In order to characterize the properties of the equilibrium as compared to the benchmark, we study the impact of setting a marginal tax. Technically we study \( \frac{\partial X}{\partial f} \bigg|_{f=0} \) with \( X \) representing any variable at the equilibrium.

In this case, we only provide the analytical expressions of prices at the second stage and optimal qualities. To determine prices at the second stage and optimal qualities, we use the profit functions defined by (6) for firm \( i \) and (21) for firm \( j \).

\(^{16}\)The Mathematica program is available to the authors upon request. It provides the analytical expressions of all variables.
8.6.1 Optimal qualities

\[
x_i^{fj} = \frac{(f - 3t)(1 + f - t)}{2(2f - 3t)(1 + 2t)} \\
x_j^{fj} = \frac{(f - 3t)(1 - 3f + 5t)}{2(2f - 3t)(1 + 2t)}
\]

From those expressions, it is straightforward to show that \(\frac{\partial x_i^{fj}}{\partial f}|_{f=0} > 0\) and \(\frac{\partial x_j^{fj}}{\partial f}|_{f=0} < 0\). Moreover, we also have \(\frac{\partial x_i^{fj}}{\partial f} = 0\) for \(f = \frac{1}{2}(3t - \sqrt{t^2 + 2t})\).

8.6.2 Equilibrium prices

Substituting \(x_i^{fj}\) and \(x_j^{fj}\) into the second stage optimal prices, we get the equilibrium prices. From which, we compute the partial derivatives. We get

\[
\frac{\partial p_i^{fj}}{\partial f}|_{f=0} = \frac{-2(1 + 4t^2)(1 + t) + (1 - t + 2t^2)}{12t(1 + 2t)^2}
\]

\[
\frac{\partial p_j^{fj}}{\partial f}|_{f=0} = \frac{2(3t - 1) + 4t^2(5t - 1) + (1 + t + 14t^2)}{12t(1 + 2t)^2}
\]

It is easy to prove that under (13) \(\frac{\partial p_i^{fj}}{\partial f}|_{f=0} < 0\) and \(\frac{\partial p_j^{fj}}{\partial f}|_{f=0} > 0\)

8.6.3 Demands and Profits

From above, we deduce the demands and profits and calculate the partial derivative wrt \(f\)

\[
\frac{\partial D_i^{fj}}{\partial f}|_{f=0} = \frac{1 + 2t}{18t^2} > 0
\]

\[
\frac{\partial \Pi_i^{fj}}{\partial f}|_{f=0} = \frac{2 + t}{3(1 + 2t)^2} > 0
\]

\[
\frac{\partial \Pi_j^{fj}}{\partial f}|_{f=0} = \frac{-(2 + 7t)}{3(1 + 2t)}
\]

It is easy to prove that under (13) \(\frac{\partial D_i^{fj}}{\partial f}|_{f=0} > 0\) and thus that \(\frac{\partial D_j^{fj}}{\partial f}|_{f=0} < 0\) as the total demand is fixed. We also have \(\frac{\partial \Pi_i^{fj}}{\partial f}|_{f=0} > 0\) and \(\frac{\partial \Pi_j^{fj}}{\partial f}|_{f=0} < 0\)
8.6.4 Health index

From the demands and optimal qualities we compute the health index and calculate the partial derivative wrt $f$

$$\frac{\partial h_{ij}^f}{\partial f}|_{f=0} = \frac{2(1-t) + 3t + 7t^2 - 1}{18t^2(1 + 2t)} > 0$$

8.6.5 Consumer surplus

We get:

$$\frac{\partial CS_{ij}^f}{\partial f}|_{f=0} = -\frac{1}{48t(1 + 2t)} < 0$$

Note that when $ff$ increases, $\frac{\partial CS_{ij}^f}{\partial f}$ can be positive.

8.6.6 Welfare

We compute the impact of taxation on SW1 and SW2 and then calculate the partial derivative wrt $f$

$$\frac{\partial SW_{1ij}^f}{\partial f}|_{f=0} = \frac{24t^2 - 1}{48t(1 + 2t)} > 0$$

$$\frac{\partial SW_{2ij}^f}{\partial f}|_{f=0} = \frac{1 + 12t + 24t^2}{48t(1 + 2t)} > 0$$

8.6.7 Optimal taxation

We have proved that $\frac{\partial SW_{1ij}^f}{\partial f}|_{f=0} > 0$ and $\frac{\partial SW_{2ij}^f}{\partial f}|_{f=0} > 0$. Then there exists a positive tax which improves welfare whether evaluated by a populist or a paternalistic social planner.

To determine the optimal tax, we need to check some conditions when optimizing the welfare function over $f$. To remain in ‘case 1’ demand, two constraints need to hold: $x_0 \geq 0$, and $x_1 \leq 1$. We also check for $x_i \geq 0$ and $x_j \leq 1$. Given the optimal choices of qualities and prices by firms, we have: $x_0 \geq 0$ $\iff f \leq \frac{4-4t+\sqrt{16-50t+52t^2}}{2}$ and $x_1 \leq 1$ $\iff f \leq \frac{8t^2-2-\sqrt{4-14t+28t^2}}{2}$. The second condition is always the most restrictive. Denoting $f_{max} = \frac{8t^2-2-\sqrt{4-14t+28t^2}}{2}$, we find that both SW1 and SW2 increase with $f \in [0, f_{max}]$.

Then, conditional on being in the ‘case 1’ demand, optimal welfare for $f = f_{max}$. We also checked that when $f = f_{max}$, we have $x_i \geq 0$ and $x_j \leq 1$. 

64
When \( f > f_{\text{max}} \), demand is defined by ‘case 3’. Given the level of taxation, the equilibrium in this area implies \( x_j = 1 \). We explore this domain numerically for the two cases \( t = 3/4 \) and \( t = 9/10 \). We find that with high level of taxation \( f > f_{\text{max}} \), the health index decreases under its benchmark value. We thus do not analyse the welfare impact as we are no longer in the case of a ‘nutritional policy’ aiming at improving health.

8.7 Extension of the Duvaleix-Tréguer et al. (2012) analysis

We use the model developed by Duvaleix-Tréguer et al. (2012) (hereafter DT) to analyse in a context of two independent characteristics (taste and healthiness) how an MQS or a linear tax on the low-quality product affects the choice of healthiness (quality) of the high-quality good (entrant). We use the same notation as DT. We also remind that the incumbent firm (denoted \( a \)) cannot modify its choice of location \( (y_a = 0) \) and healthiness \( (x_a = 0) \). In the benchmark case, DT (pp. 846 and following) find \( x_{b_{bm}}^* = \frac{(1+\gamma)\theta_2-2\alpha}{2\alpha^2} \) when \( \theta_2 > \frac{2\alpha}{1+\gamma} \), with \( x_{b_{bm}}^* \) the quality of product \( b \), \( \theta_2 \) the willingness to pay for quality of type 2 consumers, \( \alpha \) a cost parameter, and \( \gamma \) the ratio \( \theta_1/\theta_2 \) (with \( \theta_1 \) the the willingness to pay for quality of type 1 consumers).

To analyse the impact of an MQS policy, we now analyse the strategic choice of quality by firm \( b \), assuming that the incumbent firm now produces a product of quality \( x_a = mqs \). We also consider that \( \theta_2 \) is sufficiently large so that a positive \( x_b \) is the optimal choice for firm \( b \). Prices at the second-stage equilibrium are given by:

\[
p_a = \frac{1}{6}(3 + 2y_b(2 + y_b) + \alpha x_b(2 + \alpha x_b) - x_b(1 + \gamma)\theta_2 + 2\alpha x_a(2 + \alpha x_a) + x_a(1 + \gamma)\theta_2] \]

\[
p_b = \frac{1}{6}(3 + 2y_b(4 - y_b) + \alpha x_b(4 + 2\alpha x_b) + x_b(1 + \gamma)\theta_2 + 2\alpha x_a(2 + \alpha x_a) - x_a(1 + \gamma)\theta_2] \]

which extends the result by DT to the case in which \( x_a > 0 \). Using these second stage prices, we write the profit of firm \( b \) and then maximise its profit wrt \( x_b \) and \( y_b \). In particular, we find:

\[
x_b = \frac{\theta_2(1+\gamma) - 2\alpha}{2\alpha^2} \]

which does not depend on \( x_a \) and is identical to optimal choice of quality by firm \( b \) in benchmark equilibrium.
To analyse the impact of a tax policy, we now analyse the strategic choice of quality by firm $b$, assuming that the incumbent firm now faces an excise tax $t$. We also consider that $\theta_2$ is sufficiently large so that a positive $x_b$ is the optimal choice for firm $b$. Prices at the second-stage equilibrium are given by:

\[
\begin{align*}
  p_a &= \frac{1}{6} [3 + 2y_b(2 + y_b) + \alpha x_b(2 + \alpha x_b) - x_b(1 + \gamma)\theta_2 + 4t] \\
  p_b &= \frac{1}{6} [3 + 2y_b(4 - y_b) + \alpha x_b(4 + 2\alpha x_b) + x_b(1 + \gamma)\theta_2 + 2t]
\end{align*}
\]

which extends the result by DT to the case in which firm $a$ incurs a tax $t$. Using these second-stage prices, we write the profit of firm $b$ and then maximise its profit wrt $x_b$ and $y_b$. In particular, we find:

\[
x_b = \frac{\theta_2(1 + \gamma) - 2\alpha}{2\alpha^2}
\]

which does not depend on $t$. 

66